THE STUDY OF MULTIPLE ACCESS TECHNIQUES IN ULTRA WIDEBAND
IMPULSE RADIO COMMUNICATIONS

A Dissertation
Presented to
The Graduate Faculty of The University of Akron

In Partial Fulfillment
of the Requirements of the Degree
Doctor of Philosophy

Yuhua Zhao
August, 2005
ABSTRACT

In this dissertation, the system performance of multiple access schemes in Ultra Wide-band (UWB) communications is evaluated in a multipath and multiuser fading environment. Three multiple access schemes, namely Time Hopping (TH), Direct Sequence (DS) and hybrid Direct Sequence-Time Hopping (DS-TH) are investigated. The TH multiple access has been well studied in Radar communication systems. This research extends the previous studies by applying three pulse modulations techniques, including pulse position modulation, pulse shift keying and pulse amplitude modulation. The idea of the DS multiple access schemes is also generalized from the well-known Direct Sequence-Code Division Multiple Access (DS-CDMA) cellular systems to UWB radios. It is shown that the DS multiple access has the potential to reach higher data transmission rates and that TH techniques are more resistant to fading. The DS-TH multiple access schemes are proposed by combining the advantages of both TH and DS multiple access schemes. Results show that the DS-TH ultra wideband achieves better system performance while maintaining the required data transmission rate and multiple access capacity. The system performance is illustrated and examined in terms of the signal to noise plus interference ratio, bit error rate and outage probability.
ACKNOWLEDGMENTS

I would like to express my sincere appreciation to my advisor, Dr. Okechukwu C. Ugweje, for his invaluable advice, guidance, support and encouragement throughout the duration of this work, without which this work would not have been possible. I am looking forward to maintaining a continued friendship with him.

Special thanks go to Dr. Paul C. K. Lam, Associate Dean of the College of Engineering, for his help and financial support during my academic career at The University of Akron. He has been somewhat of a father figure to me. It has been a pleasure working under his supervision and his kindness and understanding will be a valuable memory in my life.

I wish to express my deep gratitude to other committee members, Dr. John Welch, Dr. Jack Durkin and Dr. Dale Mugler for their careful reading, corrections and helpful discussion during my research. Appreciation is also given to the Department of Electrical and Computer Engineering for the financial support. I want to thank all the faculty, staff members and friends in the department for their assistance, especially Dr. Tom Hartley and Ms. Gay Boden, secretary of the Department of Electrical and Computer Engineering.

My true love goes to my parents for their constant encouragement and moral support, which helps me through hard times and several challenges of completing this work. Above all, my love and admiration goes to my husband, Alex Jiaokai Jing, for his love, understanding, support and help.
TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIST OF TABLES</td>
<td>ix</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>x</td>
</tr>
<tr>
<td>LIST OF ACRONYMS</td>
<td>xiii</td>
</tr>
<tr>
<td>LIST OF SYMBOLS</td>
<td>xv</td>
</tr>
<tr>
<td>CHAPTER</td>
<td></td>
</tr>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Motivation</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Problem Statement</td>
<td>3</td>
</tr>
<tr>
<td>1.3 Importance and Contribution of Research</td>
<td>6</td>
</tr>
<tr>
<td>1.4 Dissertation Outline</td>
<td>7</td>
</tr>
<tr>
<td>II. UWB COMMUNICATIONS SYSTEM</td>
<td>8</td>
</tr>
<tr>
<td>2.1 Definition of UWB Signals</td>
<td>8</td>
</tr>
<tr>
<td>2.2 UWB Pulse Shapes</td>
<td>9</td>
</tr>
<tr>
<td>2.3 Pulse Modulation</td>
<td>12</td>
</tr>
<tr>
<td>2.3.1 Pulse Position Modulation</td>
<td>12</td>
</tr>
<tr>
<td>2.3.2 Pulse Amplitude Modulation</td>
<td>13</td>
</tr>
<tr>
<td>2.3.3 Pulse Shift Keying</td>
<td>14</td>
</tr>
<tr>
<td>2.4 Spread Spectrum Techniques</td>
<td>14</td>
</tr>
</tbody>
</table>
2.5 Generalized UWB System Model ........................................... 17
  2.5.1 Transmitter Model .................................................. 17
  2.5.2 Channel Model ....................................................... 19
  2.5.3 Receiver Model ....................................................... 21
2.6 System Performance Parameters ........................................... 24
  2.6.1 Signal-to-Noise plus Interference Ratio .......................... 24
  2.6.2 Bit Error Rate ....................................................... 25
  2.6.3 Outage Probability .................................................. 26
2.7 Diversity Combining Techniques ........................................... 26
  2.7.1 Selection Diversity .................................................. 27
  2.7.2 Maximal Ratio Combining .......................................... 29
  2.7.3 Equal Gain Combining .............................................. 30
2.8 Summary ................................................................. 31

III. TIME HOPPING UWB MULTIPLE ACCESS TECHNIQUES .............. 32
  3.1 Introduction ............................................................ 32
  3.2 Time Hopping-Pulse Position Modulation .............................. 33
    3.2.1 Transmitted TH-PPM Ultra Wideband Signals .................. 33
    3.2.2 Received TH-PPM Ultra Wideband Signals .................... 35
    3.2.3 Variance of TH-PPM Ultra Wideband Systems ................ 36
    3.2.4 Pulse Signal Properties ......................................... 40
    3.2.5 Signal-to-Noise plus Interference Ratio for TH-PPM Systems . 41
    3.2.6 Bit Error Rate for TH-PPM Systems ............................ 48

3.2.7 Outage Probability for TH-PPM Systems ................................ 57
3.3 Time Hopping-Phase Shift Keying ........................................ 61
  3.3.1 Transmitted TH-PSK Ultra Wideband Signals ......................... 61
  3.3.2 Received TH-PSK Ultra Wideband Signals ............................ 62
  3.3.3 Variance of TH-PSK Systems .......................................... 63
  3.3.4 Signal-to-Noise plus Interference Ratio for TH-PSK Systems ...... 65
  3.3.5 Bit Error Rate for TH-PSK Systems ................................... 69
  3.3.6 Outage Probability for TH-PSK Systems .............................. 75
3.4 Time Hopping-Pulse Amplitude Modulation ............................... 79
3.5 Conclusions ........................................................................ 79

IV. DIRECT SEQUENCE MULTIPLE ACCESS UWB SYSTEM ............... 81
  4.1 Introduction .................................................................. 81
  4.2 DS-PSK Ultra Wideband Signals ....................................... 82
  4.3 Variance of DS-PSK Ultra Wideband Systems ....................... 84
  4.4 System Performance for DS-PSK Ultra Wideband Communications . . 91
    4.4.1 Signal-to-Noise plus Interference Ratio ........................... 92
    4.4.2 Bit Error Rate for DS-PSK Ultra Wideband Systems .......... 96
    4.4.3 Outage Probability for DS-PSK Ultra Wideband Systems .... 101
  4.5 Conclusions .................................................................. 104

V. DIRECT SEQUENCE-TIME HOPPING MULTIPLE ACCESS .......... 106
  5.1 Introduction ............................................................... 106
  5.2 DS-TH Ultra Wideband Signals ....................................... 107
5.3 Variance of DS-TH Ultra Wideband Systems .......................... 109
5.4 System Performance for DS-TH Ultra Wideband Communications ...... 114
  5.4.1 Signal-to-Noise plus Interference Ratio .............................. 114
  5.4.2 Bit Error Rate for DS-TH Ultra Wideband Systems .................. 119
  5.4.3 Outage Probability for DS-TH Ultra Wideband Systems ............ 124
5.5 Conclusions .......................................................................... 126
VI SUMMARY AND CONCLUSIONS ............................................. 128
  6.1 Summary of Results .............................................................. 128
  6.2 Summary of Chapters ............................................................ 129
  6.3 Future Research Suggestions .................................................. 130
REFERENCES ........................................................................... 132
APPENDICES ............................................................................. 139
  APPENDIX A. DERIVATION OF THE VARIANCE OF NOISE ......... 140
  APPENDIX B. GAUSS-LAGUERRE INTEGRATION ......................... 142
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1 Pulse signal parameters for TH-PPM ultra wideband systems</td>
<td>41</td>
</tr>
<tr>
<td>3.2 Pulse signal parameters for TH-PSK ultra wideband systems</td>
<td>65</td>
</tr>
<tr>
<td>4.1 Pulse signal parameters for DS-PSK ultra wideband systems</td>
<td>91</td>
</tr>
<tr>
<td>B.1 16-point Gauss-Laguerre integration</td>
<td>142</td>
</tr>
</tbody>
</table>
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Power spectral densities of different communication techniques</td>
<td>2</td>
</tr>
<tr>
<td>2.1</td>
<td>FCC emission specification for indoor applications</td>
<td>9</td>
</tr>
<tr>
<td>2.2</td>
<td>UWB pulses in both time and frequency domains</td>
<td>10</td>
</tr>
<tr>
<td>2.3</td>
<td>Illustration of pulse modulation techniques</td>
<td>13</td>
</tr>
<tr>
<td>2.4</td>
<td>Classification of different spread spectrum systems</td>
<td>15</td>
</tr>
<tr>
<td>2.5</td>
<td>Block diagram of UWB communication system model</td>
<td>18</td>
</tr>
<tr>
<td>3.1</td>
<td>Shared channel by using TH-PPM systems</td>
<td>35</td>
</tr>
<tr>
<td>3.2</td>
<td>Average SNIR comparison for TH-PPM systems</td>
<td>44</td>
</tr>
<tr>
<td>3.3</td>
<td>Trade-off between the number of active users, $K$, and data transmission rate in TH-PPM UWB systems ($m = 1$ and $E_s/N_0 = 10$ dB)</td>
<td>46</td>
</tr>
<tr>
<td>3.4</td>
<td>Average SNIR as a function of the diversity order, $N$, in a TH-PPM UWB system</td>
<td>47</td>
</tr>
<tr>
<td>3.5</td>
<td>Effect of TH-PPM pulse shapes on the average SNIR performance</td>
<td>48</td>
</tr>
<tr>
<td>3.6</td>
<td>BER comparison for TH-PPM systems</td>
<td>52</td>
</tr>
<tr>
<td>3.7</td>
<td>Average BER of TH-PPM as a function of the number of active users, $K$</td>
<td>53</td>
</tr>
<tr>
<td>3.8</td>
<td>Average BER of TH-PPM as a function of the diversity order, $N$</td>
<td>54</td>
</tr>
<tr>
<td>3.9</td>
<td>Average BER of TH-PPM as a function of the fading parameter, $m$</td>
<td>55</td>
</tr>
<tr>
<td>3.10</td>
<td>Effect of pulse shapes on the average BER in TH-PPM systems</td>
<td>56</td>
</tr>
<tr>
<td>Section</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>---------</td>
<td>------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>3.11</td>
<td>Outage probability of TH-PPM as a function of the number of active users, $K$</td>
<td>60</td>
</tr>
<tr>
<td>3.12</td>
<td>Outage probability of TH-PPM as a function of the diversity order, $N$</td>
<td>61</td>
</tr>
<tr>
<td>3.13</td>
<td>Shared channel by using TH-PSK systems</td>
<td>63</td>
</tr>
<tr>
<td>3.14</td>
<td>Effect of pulse shapes on the average SNIR in TH-PSK systems</td>
<td>67</td>
</tr>
<tr>
<td>3.15</td>
<td>SNIR comparison for TH-PSK systems</td>
<td>68</td>
</tr>
<tr>
<td>3.16</td>
<td>SNIR comparison of PSK and PPM as a function of the number of active users and data transmission rate</td>
<td>69</td>
</tr>
<tr>
<td>3.17</td>
<td>SNIR Comparison of PSK and PPM, as a function of $N$ and $E_s/N_0$</td>
<td>70</td>
</tr>
<tr>
<td>3.18</td>
<td>BER comparison for TH-PSK systems</td>
<td>72</td>
</tr>
<tr>
<td>3.19</td>
<td>BER comparison of PSK and PPM as a function of $K$ ($m = 2$, $N = 5$ and $E_s/N_0 = 10$ dB)</td>
<td>73</td>
</tr>
<tr>
<td>3.20</td>
<td>BER comparison of PSK and PPM as a function of $N$ ($m = 2$, $K = 8$ and $E_s/N_0 = 10$ dB)</td>
<td>74</td>
</tr>
<tr>
<td>3.21</td>
<td>Comparison of the outage probability in TH-PSK systems</td>
<td>77</td>
</tr>
<tr>
<td>3.22</td>
<td>Outage probability as a function of $\varepsilon$, $K$ and $N$ in TH-PSK systems</td>
<td>78</td>
</tr>
<tr>
<td>4.1</td>
<td>Signal format in a DS-PSK ultra wideband system</td>
<td>83</td>
</tr>
<tr>
<td>4.2</td>
<td>Illustration of self-interference in a DS-PSK system</td>
<td>85</td>
</tr>
<tr>
<td>4.3</td>
<td>Illustration of the MAI effect in a DS-PSK system</td>
<td>87</td>
</tr>
<tr>
<td>4.4</td>
<td>Effect of DS-PSK pulse shapes on the average SNIR</td>
<td>93</td>
</tr>
<tr>
<td>4.5</td>
<td>Effect of fading characteristics on the average SNIR in a DS-PSK system</td>
<td>94</td>
</tr>
<tr>
<td>4.6</td>
<td>Average SNIR of DS-PSK as a function of the number of active users and data transmission rate</td>
<td>95</td>
</tr>
<tr>
<td>Section</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>---------</td>
<td>----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>4.7</td>
<td>Average BER of DS-PSK as a function of $E_s/N_0$ and $N_s$</td>
<td>98</td>
</tr>
<tr>
<td>4.8</td>
<td>Average BER of DS-PSK as a function of $N$ and $m$</td>
<td>99</td>
</tr>
<tr>
<td>4.9</td>
<td>Average BER of DS-PSK as a function of $E_s/N_0$ and $K$</td>
<td>100</td>
</tr>
<tr>
<td>4.10</td>
<td>Outage probability of DS-PSK as a function of $\varepsilon$ and $N$</td>
<td>102</td>
</tr>
<tr>
<td>4.11</td>
<td>Outage probability of DS-PSK as a function of $E_s/N_0$, $K$ and $N_s$</td>
<td>103</td>
</tr>
<tr>
<td>5.1</td>
<td>DS-TH ultra wideband signal format</td>
<td>107</td>
</tr>
<tr>
<td>5.2</td>
<td>Average SNIR of DS-TH-PSK as a function of $E_s/N_0$, $N_s$ and $N_T$</td>
<td>115</td>
</tr>
<tr>
<td>5.3</td>
<td>Average SNIR of DS-TH-PSK as a function of $E_s/N_0$ and $K$</td>
<td>116</td>
</tr>
<tr>
<td>5.4</td>
<td>Effect of the diversity order on the average SNIR</td>
<td>117</td>
</tr>
<tr>
<td>5.5</td>
<td>SNIR performance for the three multiple access schemes</td>
<td>118</td>
</tr>
<tr>
<td>5.6</td>
<td>BER performance of the three multiple access schemes ($K = 5$)</td>
<td>121</td>
</tr>
<tr>
<td>5.7</td>
<td>Trade-off between the BER and the multiple access capacity</td>
<td>122</td>
</tr>
<tr>
<td></td>
<td>($E_s/N_0 = 10 , dB$)</td>
<td></td>
</tr>
<tr>
<td>5.8</td>
<td>Effect of fading and diversity in a DS-TH ultra wideband system</td>
<td>123</td>
</tr>
<tr>
<td>5.9</td>
<td>Average BER of DS-TH as a function of $K$ and $N$</td>
<td>124</td>
</tr>
<tr>
<td>5.10</td>
<td>Outage probability comparison of three multiple access schemes</td>
<td>126</td>
</tr>
<tr>
<td></td>
<td>as a function of $K$ and $\varepsilon$</td>
<td></td>
</tr>
</tbody>
</table>
LIST OF ACRONYMS

iid Independent and identically distributed
ASK Amplitude shift keying
AWGN Additive White Gaussian Noise
BER Bit error rate
BPSK Binary pulse shift keying
CDF Cumulative density function
CDMA Code division multiple access
DS Direct sequence
EGC Equal gain combining
EIRP Equivalent isotropic radiated power
FCC Federal Communication Commission
FH Frequency hopping
HAN Home area network
IEEE Institute of Electrical and Electronics Engineers
IR Impulse radio
MAC Medium access control
MAI Multiple access interference
<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>MIP</td>
<td>Multipath intensity profile</td>
</tr>
<tr>
<td>MRC</td>
<td>Maximal ratio combining</td>
</tr>
<tr>
<td>OOK</td>
<td>On-off keying</td>
</tr>
<tr>
<td>PAM</td>
<td>Pulse amplitude modulation</td>
</tr>
<tr>
<td>PDA</td>
<td>Personal digital assistant</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability density function</td>
</tr>
<tr>
<td>PHY</td>
<td>Physical</td>
</tr>
<tr>
<td>PPM</td>
<td>Pulse position modulation</td>
</tr>
<tr>
<td>PSK</td>
<td>Pulse shift keying</td>
</tr>
<tr>
<td>RF</td>
<td>Radio frequency</td>
</tr>
<tr>
<td>SD</td>
<td>Selection diversity</td>
</tr>
<tr>
<td>SNIR</td>
<td>Signal to noise plus interference ratio</td>
</tr>
<tr>
<td>SS</td>
<td>Spread spectrum</td>
</tr>
<tr>
<td>TH</td>
<td>Time hopping</td>
</tr>
<tr>
<td>UWB</td>
<td>Ultra wideband</td>
</tr>
<tr>
<td>WLAN</td>
<td>Wireless local area network</td>
</tr>
<tr>
<td>WPAN</td>
<td>Wireless personal area network</td>
</tr>
</tbody>
</table>
LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>([x])</td>
<td>Round (x) to the nearest integer toward zero</td>
</tr>
<tr>
<td>(\beta)</td>
<td>PPM time shift</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>Instantaneous SNIR</td>
</tr>
<tr>
<td>(\overline{\gamma})</td>
<td>Average SNIR</td>
</tr>
<tr>
<td>(\delta(\cdot))</td>
<td>Delta function</td>
</tr>
<tr>
<td>(\varepsilon)</td>
<td>SNIR threshold</td>
</tr>
<tr>
<td>(\theta)</td>
<td>Pulse correlation parameter</td>
</tr>
<tr>
<td>(\mu)</td>
<td>Decay factor</td>
</tr>
<tr>
<td>(\sigma^2)</td>
<td>Variance of the interference plus noise</td>
</tr>
<tr>
<td>(\Gamma(\cdot))</td>
<td>Gamma function</td>
</tr>
<tr>
<td>(\Omega)</td>
<td>Mean power</td>
</tr>
<tr>
<td>({b_{jk}})</td>
<td>Direct sequence pseudo-random noise code</td>
</tr>
<tr>
<td>({c_{jk}})</td>
<td>Time hopping pseudo-random noise code</td>
</tr>
<tr>
<td>(d_{jk})</td>
<td>User data</td>
</tr>
<tr>
<td>dB</td>
<td>Decibel</td>
</tr>
<tr>
<td>(f_c)</td>
<td>Center frequency of the transmission</td>
</tr>
<tr>
<td>(f_{H})</td>
<td>Upper frequency of the -10 dB emission point</td>
</tr>
</tbody>
</table>
\( f_L \)  
Lower frequency of the -10 dB emission point

\( m \)  
Nakagami fading parameter

\( \text{mod}(\cdot) \)  
Reminder operator

\( n(t) \)  
AWGN noise

\( p(t) \)  
UWB pulse waveform

\( r(t) \)  
Received signal

\( s_k(t) \)  
Transmitted signal

\( s_k^*(t) \)  
Reference signal

\( t_k \)  
Transmission time difference

\( z_n \)  
Decision variable

\( A_{kl} \)  
Path gain

\( B_\Delta \)  
Fractional bandwidth

\( E_s \)  
Symbol energy

\( E_p \)  
Pulse energy

\( K \)  
Number of active users

\( L \)  
Number of multipath components

\( N \)  
Diversity order

\( N_h \)  
Number of allowed hopping slots inside a frame

\( N_s \)  
Number of pulses per symbol

\( N_T \)  
Number of hopping slots per DS chip

\( \overline{P}_b \)  
Average BER
CHAPTER I

INTRODUCTION

1.1 Motivation

Among all the competing wireless technologies currently under development, Ultra Wideband (UWB) wireless communication is undoubtedly the most interesting. A UWB communication system can be broadly classified as any communication system whose instantaneous bandwidth is much greater than the minimum required bandwidth to transmit particular information. The early UWB technique was widely used in radar and remote sensing applications [1]. In 2002, the US Federal Communication Commission (FCC) approved the commercial use of UWB technology for faster and more secure wireless transmissions and the Institute of Electrical and Electronics Engineers (IEEE) is currently working on a wireless UWB standard [2].

There are two common types of UWB communications: one is based on sending very short duration pulses to convey information and the other is based on using multiple carriers. This dissertation focuses on the first type of UWB, in which the UWB technique does not use sinusoidal carriers, but instead generates many extremely narrow pulses per second, resulting in an ultra wideband signal. Therefore, UWB is alternatively referred to as carrierless short pulses or Impulse Radio (IR) [3].
Because of the use of extremely short pulse duration signals, UWB systems have several potentially attractive advantages:

(1) Multipath immunity is a natural outcome of using short pulses [4]. Thus, the fading that is commonly experienced in mobile applications is significantly reduced or eliminated.

(2) Due to the potentially large processing gain, a UWB system is capable of supporting wireless communications at a high data rate.

(3) UWB operates within the electronic “noise” portion of the spectrum as shown in Figure 1.1. A system with ultra wide bandwidth and low power will be extremely difficult to intercept. This means that UWB will be particularly useful for communication security. In addition, the low power spectral density allows coexistence with existing communication services.

(4) One of the most interesting benefits of UWB technology is its ability to combine communications with radar applications. In a sense, a UWB system uses radar like pulses for communications, and thus could be used for locating positions, detecting intrusions and remote sensing [4].

Figure 1.1: Power spectral densities of different communication techniques.
Another benefit of UWB is its expected low cost. Unlike narrowband systems, the design of a UWB system requires relatively simple Radio Frequency (RF) electronics. This makes UWB ideal for low cost production.

UWB is desirable for data transmission, especially for the distribution of music, video, and sensor information. Currently, target environments for using UWB techniques in communications are short-range systems, such as Wireless Local Area Networks (WLANs), Wireless Personal Area Networks (WPANs) and Home Area Networks (HANs). Also, the technology might be useful for small devices, like smart cellular phones, Personal Digital Assistants (PDAs), laptops, and much more [3]. It is expected that the application of UWB will continue to grow.

1.2 Problem Statement

Although UWB technique provides new qualities and functionality, there are still several challenges in making this technology live up to its full potential. Issues of interest include:

(1) Propagation measurements and channel modeling [5]-[12]. To perform system level engineering, the UWB technique must consider its propagation characteristics. Multi-path signals should be resolvable quickly to provide enough energy in order to recover the information signals. Thus, a valid statistical channel model is an important prerequisite for UWB system design and implementation. Proper analysis of the measured data will enable the development of a consistent channel model for system evaluation and simulation, in particular with respect to channel parameters such as the delay profiles, fading statistics and temporal and spatial correlations [5]-[12].
Multiple access modulation techniques for UWB transmission [13]-[20]. With high
user densities, the performance of the network should be interference limited, and mul-
tiuser interference cancellation is necessary. Thus achieving better system performance
necessitates the study of multiple access schemes and receiver designs [21]-[34].

Signal shapes design [35]-[41]. UWB differs from conventional communications in
that the signal shape might be changed due to the effect of channel fading and antenna
radiation pattern. Therefore, the design of pulse shapes has received more and more
attention.

Timing acquisition and reception of ultra short pulses [42]-[47]. The problem is mainly
due to the low received signal power, which forces the acquisition system to process
the signal over long periods of time before getting a reliable estimate of the timing
of the signal. Hence there is a need to develop more efficient acquisition schemes by
taking into account the signal and channel characteristics. In addition, due to the ex-
tremely short pulses, the need for highly accurate synchronization between transmitter
and receiver must be considered.

Coexistence of a UWB system with other systems [48]-[52]. UWB radios, operating
with extremely large bandwidths, must coexist with many other interfering narrow-
band signals. At the same time, these narrowband systems must not suffer intolerable
interference from the UWB radios. Thus, another concern is added to the design of the
UWB systems.

The implementation issues of UWB systems in network environments [53], [54]. Even-
tually, UWB products must be applied to a network environment. The network should
be self-organized and dynamic so that clients can join or leave at any time. Issues about
both Physical (PHY) and Medium Access Control (MAC) layers must be addressed and related protocols should be designed.

In this dissertation, UWB multiple access techniques are studied over a multipath indoor wireless channel, thus addressing item (2) for application purposes.

Multipath propagation occurs when an RF signal takes different paths from a transmitter to a receiver due to reflection, refraction and scattering [55]. A portion of the signal may go directly to the destination, and some of the signal will encounter delays and travel longer paths to the receiver. This can result in Inter-Symbol Interference (ISI), which is a critical phenomenon in short-range indoor environments. For the signal to survive in a fading environment, one solution is to increase the transmitted power. However, this is not a viable solution due to the limitation on power mandated by law. In cellular systems, another solution might be the use of RAKE receivers and diversity techniques [55], which provide a more practical and feasible solution for performance improvement. This work characterizes several kinds of diversity techniques for UWB systems.

The multiple access capacity is one of the most important considerations in any multi-user wireless communication system. Modulation techniques of IR systems have been well studied in radar systems for decades [5]. However, for UWB radios, modulation and coding schemes need to be developed. To achieve better multiple access capability and highly resolved multipath, Spread Spectrum (SS) communication techniques are employed in cellular communications, including Time Hopping (TH), Frequency Hopping (FH) and Direct Sequence (DS) [6]. This dissertation investigates SS techniques for applications in UWB systems. The various modulation schemes for currently existing UWB systems are
also presented along with some new systems proposed in this dissertation. In addition, the system performance in a multipath and multiuser fading channel is evaluated.

1.3 Importance and Contribution of Research

The contribution of this research can be summarized as follows:

(1) Analysis of UWB systems with a generalized channel model. In this dissertation, the propagation channel is modeled as a multipath channel, which is known to provide a close match to experimental data [55]. Moreover, the Nakagami distribution is generalized since it can approximate several fading conditions, such as Gaussian, Rayleigh and Ricean distributions.

(2) Application of three different diversity combiners in UWB systems. The diversity techniques considered include Selection Diversity (SD), Maximal Ratio Combining (MRC) and Equal Gain Combining (EGC).

(3) Presentation of a new multiple access scheme for UWB communication systems. Beside Pulse Position Modulation (PPM), two more pulse modulation techniques for Time Hopping (TH) multiple access UWB systems, Pulse Amplitude Modulation (PAM) and Pulse Shift Keying (PSK), are applied. The Direct Sequence-Time Hopping (DS-TH) multiple access scheme with the relative receiver design is proposed and analyzed in this dissertation.

(4) Numerical comparison among existing and proposed multiple access schemes, such as Time Hopping-Pulse Position Modulation (TH-PPM), Time Hopping-Pulse Shift Keying (TH-PSK), Direct Sequence-Pulse Shift Keying (DS-PSK) and DS-TH. The system performance of each scheme is analyzed under a multipath fading environment.
1.4 Dissertation Outline

In Chapter II, the fundamentals of UWB communications are described. Several signal shapes are expressed in both time and frequency domains. This is followed by the introduction of pulse modulation techniques, and then the proposed system model, including the transmitter, channel and receiver model. The expressions of UWB system performance parameters are also defined, namely Signal-to-Noise plus Interference Ratio (SNIR), Bit Error Rate (BER), and outage probability. Finally, the introduction of diversity combiners is presented.

From Chapter III to Chapter V, each chapter covers one proposed multiple access scheme. The review of each scheme is presented first. Then basic modulation/demodulation techniques and mathematical expressions are given for performance analysis. Analytical expressions for different diversity combiners and signal waveforms are deduced. Finally, the average SNIR, BER and outage probability performances are plotted as the function of the data transmission rate, the number of active users, the multipath components, the fading parameters and the signal power.

Finally, Chapter VI concludes this dissertation with a summary of the work completed and a list of suggested future research topics.
CHAPTER II

UWB COMMUNICATIONS SYSTEM

2.1 Definition of UWB Signals

A UWB signal is defined as any signal whose fractional bandwidth is greater than 25% or occupies at least 500 MHz in the allocated spectrum of the 3.1-10.6 GHz band [57]. The fractional bandwidth as defined by the FCC is given by

$$B_{\Delta} = \frac{2(f_H - f_L)}{f_H + f_L},$$

(2.1)

where $f_H$ and $f_L$ are the upper and lower frequencies of the -10 dB emission point respectively. Furthermore, the center frequency of the transmission could be defined as the average of the upper and lower -10 dB points, i.e., $f_c = (f_H + f_L) / 2$.

In order to protect both UWB communication systems and existing services against interference, emission limits were specified for UWB devices. For indoor communication systems, the Equivalent Isotropic Radiated Power (EIRP) density must remain below -41.3 dBm/MHz, as shown in Figure 2.1.

It is noticed that this regulation is based on a frequency band with power limitations, not on the type of data modulation and multiple access schemes. This brings more flexibility in the use of UWB techniques in wireless communication systems.
2.2 UWB Pulse Shapes

In UWB communications, information is contained in trains of extremely short pulses, typically from a few tens of picoseconds to a few nanoseconds [4]. Since UWB systems use the baseband technology, the choice of pulse shapes will affect the system performance. Several possible pulse shapes, with the signal amplitude $A$, are discussed in this section.

(a) Rectangular Pulse Shape

The simple rectangular shaped signal is shown in Figure 2.2 (a), which is mathematically defined as [58]

$$p(t) = \begin{cases} A, & -\frac{T_p}{2} \leq t \leq \frac{T_p}{2} \\ 0, & \text{elsewhere,} \end{cases}$$  \hspace{1cm} (2.2)

where $T_p$ is the pulse width in picoseconds and the pulse $p(t)$ is centered at $t = 0$. The Fourier transform of $p(t)$, denoted as $P(f)$ is the well known sinc function, given by

$$P(f) = \frac{AT_p \sin(\pi T_p f)}{\pi T_p f}.$$  \hspace{1cm} (2.3)
For a sinc function, the main lobe is between $f = -1/T_p$ and $f = 1/T_p$, and there are energy lobes outside.

![UWB pulses in both time and frequency domains.](image)

Figure 2.2: UWB pulses in both time and frequency domains.

(b) Gaussian Pulse Shape

A Gaussian pulse, as shown in Figure 2.2 (b), can be represented by

$$p(t) = A \exp\left(-\frac{t}{t_m}\right)^2,$$  \hspace{1cm} (2.4)

\hspace{1cm}

10
where $t_m$ is a time-scaling factor, which determines the width of the pulse. Its frequency representation is also a Gaussian shape given by

$$P(f) = A t_m \sqrt{\pi} \exp \left[ - (\pi t_m f)^2 \right]. \quad (2.5)$$

(c) Gaussian Monocycle Pulse Shape

The first derivative of a Gaussian pulse, called a Gaussian monocycle, is also commonly used, as shown in Figure 2.2 (c). A Gaussian monocycle pulse is described as

$$p(t) = -\frac{2At}{t_m^2} \exp \left( -\frac{t}{t_m} \right)^2, \quad (2.6)$$

and the corresponding frequency domain representation is given by

$$P(f) = A t_m \sqrt{\pi} (j2\pi f) \exp \left[ - (\pi t_m f)^2 \right]. \quad (2.7)$$

(d) Gaussian Doublet Pulse Shape

The second derivative of a Gaussian signal is called a Gaussian doublet, as shown in Figure 2.2 (d). In the time domain, it is defined by

$$p(t) = \frac{2A}{t_m^2} \left( 1 - \frac{2t^2}{t_m^2} \right) \exp \left( -\frac{t}{t_m} \right)^2, \quad (2.8)$$

and in the frequency domain, the signal has the form

$$P(f) = A t_m \sqrt{\pi} (j2\pi f)^2 \exp \left[ - (\pi t_m f)^2 \right]. \quad (2.9)$$

The rectangular and Gaussian pulses can be compared in terms of their design parameters by making the 10 dB bandwidths in the frequency domain exactly equal. Clearly, the main lobes between 10 dB points are nearly identical, but the side lobes are quite different: Gaussian pulses have the lower side lobe energy, but rectangular pulses show the opposite.
Even though a rectangular pulse contains more energy outside of the main lobe, it is still one of the popular shapes for the system performance analysis due to its simplicity.

It must be pointed out that many more different pulse shapes may be used for UWB applications [35]-[41]. The choice of pulse shapes is usually driven by the system design and application requirements. Gaussian pulse shapes are chosen since they are relatively easy to generate [58]. The most important factor to use the Gaussian waveform is the effect of filtering due to antennas at both transmitters and receivers. After being filtered by the transmitter antenna, the output signal is the first derivative of the input signal, while the same effect occurs at the receiver [59]. Therefore, a Gaussian doublet is always considered as the received pulse shape if a Gaussian pulse is used at the input of the transmitter antenna.

2.3 Pulse Modulation

One pulse by itself does not contain a lot of information. User data needs to be modulated onto a sequence of pulses, or a pulse train. In UWB systems, information can be encoded in a variety of ways, including amplitude, time and phase modulation. Three popular modulation schemes used for UWB communications are Pulse Position Modulation (PPM), Pulse Amplitude Modulation (PAM) and Phase Shift Keying (PSK). These modulation techniques are briefly discussed.

2.3.1 Pulse Position Modulation

Pulse Position Modulation (PPM) is based on the principle of encoding information with two or more positions in time, with references to the nominal pulse position. For example, in a binary system, as shown in Figure 2.3, a pulse transmitted at the nominal position represents a bit 0, and a pulse transmitted after the nominal position represents
a bit 1. This is achieved by time shifting, which is typically a fraction of a nanosecond and less than the time duration between nominal positions to avoid interference between impulses.

![Diagram of pulse modulation techniques](image)

Figure 2.3 Illustration of pulse modulation techniques.

### 2.3.2 Pulse Amplitude Modulation

Pulse Amplitude Modulation (PAM) is based on the principle that the amplitude of the pulses is encoded by data. Digital PAM is also called Amplitude Shift Keying (ASK), alternatively referred to as On-Off Keying (OOK) for two level PAM. Figure 2.3 shows a two-level amplitude modulation. Bit 1s and bit 0s are transmitted by different amplitudes.
2.3.3 Pulse Shift Keying

Pulse Shift Keying (PSK) is based on modulating information by phase shifting. The Binary PSK is also referred to as bi-phase modulation. In this case, the modulated signal shifts the phase of pulses, for example, at zero degrees for transmitting bit 1s and 180 degrees for transmitting bit 0s.

Among the three pulse modulation techniques, PPM and PSK could be used in UWB systems. The advantages of PPM mainly arise from its simplicity and the ease with which the time shift may be controlled [14]. Fine time control is necessary for UWB systems due to the extremely short pulses. The benefit of using PSK is its power efficiency and reduced jitter requirements [59]. PAM might not be the preferred method for most short-range communications due to the extremely low signal power.

2.4 Spread Spectrum Techniques

It is commonly believed that UWB communication systems will be required for future generation wireless communication systems. Thus, the successful deployment of the UWB technology strongly depends on efficient multiple access techniques. In cellular communication systems, spread spectrum techniques gain their popularity in dense Multiple Access Interference (MAI) propagation channels. Spread spectrum (SS) is an RF communication modulation technique in which the baseband signal bandwidth is intentionally spread over a larger bandwidth and the spread signal is composed of the information signal and the spreading sequence. In contrast to narrowband communications, spread spectrum techniques are able to communicate through environments of severe interference. The ro-
bustness of SS in interference-prone environments makes the technology suitable for both military and commercial applications [58].

Different spread spectrum techniques are available. Figure 2.4 shows a classification of spread spectrum techniques; namely, Direct Sequence Spread Spectrum (DSSS), Frequency Hopping Spread Spectrum (FHSS), Time Hopping Spread Spectrum (THSS) and the hybrid spread spectrum.

![Spread Spectrum Techniques Diagram]

Figure 2.4: Classification of different spread spectrum systems.

(a) Direct Sequence Spread Spectrum

Direct Sequence Spread Spectrum (DSSS) spreads the energy of the signal over a large bandwidth. Thus, the energy per unit frequency is reduced and the interference is decreased. This allows multiple signals to share the same frequency band. To an unintended receiver, DSSS signals appear as low-power wideband noise and are rejected.

DSSS combines the data stream with a high-speed digital code. Each data bit is mapped into a common pattern of bits (also called chips), named a Pseudo-random Noise (PN) code [60], and known only to the transmitter and the corresponding receiver. A key parameter for DSSS systems is the number of chips per bit, called the processing gain. If there is an interference or jammer in the same band, it will be spread out by a factor
equivalent to the processing gain. The chipping code is a redundant bit pattern for each bit that is transmitted, which increases the signal’s resistance to interference. A key feature of DSSS is that multiple access capability can be achieved without synchronization between different transmitters. But the transmitter and receiver must be synchronized to operate properly.

(b) Time Hopping Spread Spectrum

Time Hopping Spread Spectrum (THSS) modulates a signal by using a PN sequence to control the time of a transmission. Within a THSS system, the time axis is virtually divided into frames and slots within the frame. Only one slot is used out of \( n \) possible slots within one frame for a single user. Thus, the sending data rate is \( n \) times higher in contrast to the situation where the whole frame is used. MAI can be minimized if proper coordination between transmitters can be achieved. If more than one transmitter use the same time slot, the receivers will not be able to detect either of the signals correctly. For such cases, error correction schemes must be applied.

(c) Frequency Hopping Spread Spectrum

Frequency Hopping Spread Spectrum (FHSS) spreads the signal by using a different carrier frequency at different times. The change of frequency is based on a pattern known to both the transmitter and the receiver and the process of changing the carrier from one frequency to another (i.e., a hopping) is facilitated by the PN sequence. During a small amount of time, the signal at the given frequency is constant. However, the sequence of different channels, which determines the hopping pattern, is pseudo random. Since it is difficult to predict the next frequency at which a system will transmit data, FHSS system is secure against interception.
A comparison of the three basic spread spectrum techniques shows that FHSS and THSS do not spread the signal directly unlike DSSS, but they use spreading code sequences to determine the hopping sequence or the timing sequence. Furthermore, hopping is usually more costly and more complicated because it needs extra circuits for synchronizing in both time and frequency. Between the two hopping schemes, the application of THSS is much simpler than that of FHSS. FHSS has the shortest acquisition time among these techniques, THSS has the highest bandwidth efficiency, and DSSS shows most effective throughput in the time domain [59].

Obviously, combining two or more pure spread spectrum techniques can offer a combination of their advantages, but it may increase the complexity of the transmitter and receiver. In this research, the DS-TH technique is applied to UWB communication systems. This is a hybrid technique combining the techniques of DSSS and THSS.

2.5 Generalized UWB System Model

The block diagram of a generalized UWB system is illustrated in Figure 2.5. Throughout this dissertation, it is considered that multiple users’ UWB signals are simultaneously transmitted through a multipath fading channel. At the receiver, the desired signal is corrupted by MAI and noise.

2.5.1 Transmitter Model

In the system model, \( K \) simultaneous users transmit UWB signals through a multipath channel. User binary data can be from any application, such as an e-mail client, a web browser, a personal digital device or a digital stream from a DVD player [59].
Figure 2.5: Block diagram of UWB communication system model.
After coding, the binary data are then mapped from bits to symbols. Our concern is that of mapping symbols to analog pulse shapes, and then generating and transmitting pulses through the antenna. This requires precise timing circuits and optional amplifier circuits, although they are not shown in Figure 2.5. Basically, each transmitter generates a pulse train modulated by the data stream. In this case, multiuser information transmissions can be achieved by pulse modulation and SS techniques.

In general, the transmitted symbol of the \( k \)th user can be expressed as

\[
s_k(t) = \sqrt{\frac{E_s}{N_s}} \sum_{j=0}^{N_s-1} F(j, k) p_{tr}[t - t_k - G(j, k)],
\]

where \( N_s \) is the number of pulses per symbol; \( E_s \) is the symbol energy; \( t \) is the first user’s transmitter clock time; \( t_k \) is the relative transmission time difference compared to the first user with \( t_1 = 0 \); and \( p_{tr} \) represents the transmitted waveform based on the particular pulse shape. In addition, \( F(j, k) \) and \( G(j, k) \) are used to generalize modulation functions, which are varied from each multiple access scheme and will be derived in the following chapters. For simplicity, we define \( P_{jk}^{tr}(t) = F(j, k) p_{tr}[t - t_k - G(j, k)] \), such that (2.10) can be written as

\[
s_k(t) = \sqrt{\frac{E_s}{N_s}} \sum_{j=0}^{N_s-1} P_{jk}^{tr}(t).
\]

### 2.5.2 Channel Model

In this study, a discrete channel model is adopted [51], [55], by assuming that the minimum path resolution time is equal to the pulse width, \( T_p \). Therefore, multipath components arrive at some integer multiple of a minimum path resolution time. In addition, the same number of multipath components are chosen for each user, denoted as \( L \), with the
assumption that multipath components are mutually uncorrelated. The impulse response of the channel is then given by

\[ h_k(t) = \sum_{l=0}^{L-1} A_{kl} \delta(t - lT_p), \]  

(2.12)

where the subscript \( kl \) indicates the \( k \)th user and \( l \)th path; \( \delta(\cdot) \) is the delta function; and \( A_{kl} \) is the path gain, assumed to be Nakagami distributed according to \((A_{kl}, m_{kl}, \Omega_{kl})\), with the Probability Density Function (PDF) given by [61],

\[ f_{A_{kl}}(x) = \frac{2}{\Gamma(m_{kl})} \left( \frac{m_{kl}}{\Omega_{kl}} \right)^{m_{kl}} x^{2m_{kl}-1} \exp\left( -\frac{m_{kl}x^2}{\Omega_{kl}} \right), \]  

(2.13)

where \( \Gamma(\cdot) \) is the gamma function, defined as \( \Gamma(m) = \int_0^\infty x^{m-1} \exp(-x)dx \); and \( m \geq \frac{1}{2} \) is the Nakagami signal fading parameter, indicating the severity of fading. The Nakagami distribution may be regarded as a general statistical model of the wireless channel fading gain, which includes those modeled by the Rayleigh distribution \((m = 1)\), the one-sided Gaussian distribution \((m = \frac{1}{2})\), the lognormal distribution, Ricean distribution and non-fading condition \((m \to \infty)\) [61]. In addition, \( \Omega_{kl} \) represents the mean power with \( \Omega_{kl} = E[A_{kl}^2] \). If the uniformly distributed Multipath Intensity Profile (MIP) is assumed for each user over each multipath component, then

\[ \Omega_{kl} = \Omega, \]  

(2.14)

Similarly, under the assumption of the exponential MIP, it is obtained that

\[ \Omega_{kl} = \Omega \exp(-\mu l), \]

(2.15)

\[ \sum_{l=0}^{L-1} \Omega_{kl} = \frac{1 - \exp(-\mu L)}{1 - \exp(-\mu)}, \]
where $\mu$ is the decay factor. That means the mean power $\Omega_{kl}$ has exponential relation with the first arrived signal version $\Omega$ for each user by the decay factor $\mu$. In this dissertation, we assume the same channel model for each UWB system with $L = 20$, $\Omega = 5$ dB and $\mu = 0.1$ [12].

The UWB indoor channel model is assumed to be a linear time-invariant system if the transmitter and receiver are static and no motion takes place in the channel [29]. This assumption has been used successfully in mobile radio applications. Thus, the received signal due to user $k$ is given by

$$y_k(t) = s_k(t) \otimes h_k(t) = \sum_{l=0}^{L-1} A_{kl}s_k(t - lT_p),$$

(2.16)

where $\otimes$ denotes the convolution operator.

2.5.3 Receiver Model

At the receiver, the signals of all $K$ users with $L$ path components are summed, such that the total received signal is given by

$$r(t) = \sum_{k=1}^{K} y_k(t) + n(t)$$

$$= \sqrt{\frac{E_s}{N_s}} \sum_{k=1}^{K} \sum_{l=0}^{L-1} \sum_{j=0}^{N_s-1} A_{kl}P_{jk}^{rec}(t - lT_p) + n(t),$$

(2.17)

where $n(t)$ is Additive White Gaussian Noise (AWGN) with two-sided power spectral density $\frac{N_0}{2}$ watts/Hz. Similarly, the notation $P_{jk}^{rec}(t)$ stands for

$$P_{jk}^{rec}(t) = F(j, k)p_{rec}[t - t_k - G(j, k)]$$

(2.18)

where $p_{rec}(\cdot)$ is the received pulse train, modified due to the effect of antenna systems and the propagation channel.
A variety of receiver structures might be applied to demodulate UWB signals. To track multiple pulses caused by multipath in a wireless channel, the basic concept of RAKE receivers is used with \( N \) taps (or branches), \( N \leq L \). As discussed in Chapter I, the effect of multipath brings multiple copies of the transmitted pulse. By employing \( N \) delayed versions of the template waveform \( s_1^*(t) \), the receiver provides \( N \) replicas of the same transmitted signals. Thus, the multipath components are resolved and combined to obtain a diversity advantage.

The major drawbacks of using a RAKE receiver are the increased complexity of circuits and channel estimation. For example, a typical multipath channel can have up to approximately 70 resolvable path components [55]. Even if it is possible for a RAKE receiver to provide so many taps, it could increase the complexity of the design. On the other hand, each multipath undergoes different channel distortion. Hence, it might become difficult to estimate the channel parameters, so degradation in performance could be observed due to imperfect channel estimation and synchronization.

The optimal receiver technique used in UWB systems is a pulse correlator. This is implemented by using the reference waveforms \( s_1^* \) correlated with the received signal \( r(t) \). For simplicity, it is assumed that the signal is stable during each symbol period, \( T_b \) and the reference signal is perfectly synchronous with the desired signal, which is the signal from the user 1. Thus, in a binary system, the reference signal of the \( n \)th tap may be written as

\[
s_1^n(t) = \sum_{i=0}^{N_s-1} P_{i1}^* (t),
\]

(2.19)

where the energy of \( s_1^n(t) \) is normalized to one, i.e., \( \int_{-\infty}^{\infty} [s_1^n(t)]^2 dt = 1 \). The structure of developing delayed template waveforms is also shown in Figure 2.5. The correlation is in
fact the multiply-and-integrate process, which is a measure of the relative time position of the received pulses and occurs over the duration of the pulse. The output of this process yields a DC voltage. Since UWB signals are below the noise level, it is difficult to detect a single UWB pulse. However, by adding together multiple pulses in a symbol, it becomes possible to recover the transmitted signals with higher confidence. This process is known as pulse integration [14].

At each diversity branch, the received signal is correlated by the reference signal, such that the output of the $n$th branch becomes

$$z_n = \int_0^{T_b} r(t)s_1^*(t-nT_p)dt$$

$$= \sqrt{\frac{E_s}{N_s}} \sum_{k=1}^{K} \sum_{l=0}^{L-1} \sum_{j=0}^{N_s-1} \sum_{i=0}^{N_s-1} A_{kl} \int_0^{T_b} P_{rec}^k(t-lT_p)P_{i1}^* (t-nT_p)dt$$

$$+ \sum_{i=0}^{N_s-1} \int_0^{T_b} n(t)P_{i1}^* (t-nT_p)dt.$$  \hspace{1cm} (2.20)

Defining $R_{xy}(\tau) = \int_{-\infty}^{\infty} x(t+\tau)y(t)dt$ as a cross-correlation function, (2.20) becomes

$$z_n = A_{1n} \sqrt{\frac{E_s}{N_s}} \sum_{j=0}^{N_s-1} R_{P_{j1}^*P_{j}^*}(0)$$

$$+ \sqrt{\frac{E_s}{N_s}} \sum_{l=0, l \neq n}^{L-1} \sum_{j=0}^{N_s-1} \sum_{i=0}^{N_s-1} A_{1l} R_{P_{j1}^*P_{i1}^*}(nT_p-lT_p)$$

$$+ \sqrt{\frac{E_s}{N_s}} \sum_{k=2}^{K} \sum_{l=0}^{L-1} \sum_{j=0}^{N_s-1} \sum_{i=0}^{N_s-1} A_{kl} R_{P_{j1}^*P_{i1}^*}(nT_p-lT_p)$$

$$+ \sum_{i=0}^{N_s-1} R_{nP_{i1}^*}(nT_p).$$  \hspace{1cm} (2.21)

where the first, second, third and fourth terms in (2.21) denoted as $z_{n1}$, $z_{n2}$, $z_{n3}$, and $z_{n4}$, represent the desired signal, the self-multipath interference, MAI, and AWGN terms respectively. It is also noted that the desired signal is obtained by the perfect synchronization,
i.e., \( n = l \) and \( i = j \). The decision variable at the output of the diversity combiner then depends on the use of diversity combiners.

2.6 System Performance Parameters

2.6.1 Signal-to-Noise plus Interference Ratio

The Signal-to-Noise plus Interference Ratio (SNIR) is the ratio of wanted information (the signal) to unwanted information (noise plus interference). It is a convenient performance measure of a communication system, commonly expressed as a logarithmic ratio in decibels (dBs). In this dissertation, the SNIR is obtained under the assumption that the multiple access interference and multipath interference are zero-mean Gaussian independent Random Variables (RVs), while the average SNIR is the ratio of the square mean of the desired signal to the interference pulse noise variance.

By the Gaussian approximation technique [55], [62], at the output of the \( n \)th RAKE branch, the instantaneous SNIR is given by

\[
\gamma_n = \frac{z_n^2}{\sigma_n^2},
\]

(2.22)

where \( \sigma_n^2 \) is the variance of the interference plus noise, defined as

\[
\sigma_n^2 = \text{var} [z_{n2}] + \text{var} [z_{n3}] + \text{var} [z_{n4}],
\]

(2.23)

where \( \text{var} [x] \) denotes the variance of \( x \). The instantaneous SNIR could be written as

\[
\gamma_n = A_{1n}^2 R_n,
\]

(2.24)

with
Thus, the average received tap symbol SNIR becomes [55, pp. 278]

\[ R_n = \frac{E_s N_s R_{prev_1}^2 p_{j_1}^* (0)}{\sigma_n^2}. \]  

(2.25)

where \( f_{A_{1n}}(\cdot) \) is the PDF of \( A_{1n} \), which is Nakagami distributed, as shown in (2.13). The average SNIR at the output of the diversity combiner depends on the use of diversity techniques.

### 2.6.2 Bit Error Rate

In communication systems, the Bit Error Rate (BER) at the receiver is an important performance measure. The BER is used to evaluate the quality of a communication system by comparing messages before and after transmission. It is defined as the rate of errors among the total number of bits transferred. In a binary system, the bit error occurs when a transmitted bit 1 is detected as bit 0, or the transmitted bit 0 is detected as bit 1. The error rate depends on the modulation and detection format. If the sum of noise plus interference is assumed as a Gaussian RV, and the probability of transmitting both bit 1 and bit 0 is identical, the conditional BER becomes

\[ P_b(\gamma) = \frac{1}{2} \Pr (z < 0|\text{bit 0}) + \frac{1}{2} \Pr (z \geq 0|\text{bit 0}) , \]  

(2.27)

where \( \Pr (x|a) \) represents the conditional probability.

To account for the effect of multipath fading, the conditional BER given above is averaged over the PDF at the output of the diversity combiner. This is achieved by evaluating
the integral specified by

\[ P_b = \int_0^\infty P_b(x)f_\gamma(x)dx. \]  \hfill (2.28)

where \( f_\gamma(\cdot) \) is the PDF of the SNIR \( \gamma \); and \( P_b(\cdot) \) is defined in (2.27).

2.6.3 Outage Probability

Another important performance criterion used in wireless communications is the outage probability. In order to correctly detect a transmitted signal, the received signal must be greater than a specified threshold. However, due to the multipath effect, interference and noise, the signal is not always large enough for the signal detection. There will always be occasions when the instantaneous signal gain is not large enough for satisfactory reception, even though the mean signal strength is adequate. Therefore, the received signal must satisfy the minimum signal power threshold or the minimum error probability. In this study, the outage probability is defined as the probability of failing to achieve a minimum SNIR, \( \varepsilon \) \cite{63}, e.g.

\[ P_{out} = \Pr (\gamma < \varepsilon) = \int_0^\varepsilon f_\gamma(x)dx. \]  \hfill (2.29)

2.7 Diversity Combining Techniques

Diversity is a powerful communication receiver technique that improves the reliability of wireless communication systems by having more than one copy of some desired signal to select from. In diversity combining schemes, the received signal from several transmission paths, all carrying the same information, are combined to improve some decision variables used in the detection process. Diversity combining techniques could be based on space (antenna), frequency, angle of arrival, polarization, and time of signal reception. In
this study, three diversity combining techniques are presented, namely selection diversity, maximal ratio combining and equal gain combining.

2.7.1 Selection Diversity

With Selection Diversity (SD), the branch with the best desirable signal is selected, and the weaker ones are discarded. Among the combining techniques, SD is very attractive because of its implementation simplicity. The selection process may be directed to select the branch with the largest signal power as the most desirable reception. However, this is difficult to implement in practice since it requires the separation of the desired signal power from interference pulse noise power. Instead, a more practical selection process may be applied, by which the signal with the highest SNIR is selected.

It is assumed that the channel does not change significantly over one symbol period and the tap selection is continuous. For \( N \) diversity branches, the diversity combiner’s output SNIR is given by [55]

\[
\gamma^{SD} = \max \left( \gamma_0, \gamma_1, \ldots, \gamma_n, \ldots, \gamma_{N-1} \right),
\]

where \( \gamma_n \) is the instantaneous SNIR at the \( n \)th branch, defined in (2.26).

Since the signal amplitude is assumed as a Nakagami RV according to \((A_{1n}, m_{1n}, \Omega_{1n})\), the instantaneous SNIR \( \gamma_n \) is also Nakagami distributed with the PDF shown as

\[
f_{\gamma_n=\gamma_n}(x) = f_{A_{1n}^2 R_n}(x) = f_{A_{1n}^2 \left( \frac{x}{R_n} \right)} \left( \frac{1}{R_n} \right) = \frac{1}{\Gamma(m_{1n})} \left( \frac{m_{1n}}{\Omega_{1n} R_n} \right)^{m_{1n}} x^{m_{1n}-1} \exp \left( -\frac{m_{1n}x}{\Omega_{1n} R_n} \right).
\]

The corresponding Cumulative Density Function (CDF) can be shown as
\[
F_{\gamma_n}(x) = \int_0^x f_{\gamma_n}(y) \, dy
= \int_0^x \frac{1}{\Gamma(m_{1n})} \left( \frac{m_{1n}}{\Omega_{1n} R_n} \right)^{m_{1n}} y^{m_{1n}-1} \exp \left( -\frac{m_{1n} y}{\Omega_{1n} R_n} \right) \, dy
= \frac{1}{\Gamma(m_{1n})} \int_0^{\frac{m_{1n} x}{\Omega_{1n} R_n}} Y^{m_{1n}-1} \exp (-Y) \, dY
= \frac{G \left( m_{1n}, \frac{m_{1n} x}{\Omega_{1n} R_n} \right)}{\Gamma(m_{1n})},
\]

(2.32)

where \( G(m, x) = \int_0^x y^{m-1} \exp (-y) \, dy \) is the incomplete gamma function [65]. For real integer values of Nakagami \( m \)-parameter, \( G(m, x) \) can be written as [65]

\[
G(m, x) = \Gamma(m) \left[ 1 - \exp (-x) \sum_{i=0}^{m-1} \frac{x^i}{i!} \right].
\]

(2.33)

If the branches are independent, the CDF of \( \gamma_{SD} \) becomes [55]

\[
F_{\gamma_{SD}}(x) = \Pr \left[ \gamma_0 \leq x, \gamma_1 \leq x, \cdots, \gamma_{N-1} \leq x \right]
= \prod_{n=0}^{N-1} F_{\gamma_n}(x).
\]

(2.34)

Differentiating (2.34) and subsisting (2.32), the PDF of \( \gamma_{SD} \) can be written as

\[
f_{\gamma_{SD}}(x) = \left[ F_{\gamma_{SD}}(x) \right]'
= \sum_{n=0}^{N-1} f_{\gamma_n}(x) \prod_{i=0, i \neq n}^{N-1} F_{\gamma_i}(x)
= \sum_{n=0}^{N-1} \left( \frac{m_{1n}}{\Omega_{1n} R_n} \right)^{m_{1n}} \left( \frac{x^{m_{1n}-1}}{\Gamma(m_{1n})} \right) \exp \left( -\frac{m_{1n} x}{\Omega_{1n} R_n} \right)
\times \prod_{i=0, i \neq n}^{N-1} \frac{G \left( m_{1i}, \frac{m_{1i} x}{\Omega_{1i} R_i} \right)}{\Gamma(m_{1i})}.
\]

(2.35)
2.7.2 Maximal Ratio Combining

With Maximal Ratio Combining (MRC), the diversity branches are weighted, co-phased and combined in order to achieve the maximal signal strength. Among all the linear diversity techniques, it provides the optimum detection for the received signal, since all the multipath diversity branches are taken into consideration.

When the MRC is applied, at the output of the combiner the decision variable is given by,

\[ z^{MRC} = \sum_{n=0}^{N-1} w_n z_n, \tag{2.36} \]

where \( w_n \) is the weight function. Hence, the instantaneous SNIR at the output of combiner becomes

\[ \gamma^{MRC} = \frac{\left( \sum_{n=0}^{N-1} w_n z_n \right)^2}{\text{var} \left( \sum_{n=0}^{N-1} w_n (z_n + z_n 2 + z_n 3 + z_n 4) \right)} \]

\[ = \frac{\left( \sum_{n=0}^{N-1} w_n z_n \right)^2}{\sum_{n=0}^{N-1} w_n^2 \sigma_n^2}, \tag{2.37} \]

where \( \sigma_n^2 \) is the variance of noise plus interference defined in (2.23). With the optimum weight \( w_n = \frac{z_n}{\sigma_n^2} \), \( \gamma^{MRC} \) becomes the sum of the SNIR, given by

\[ \gamma^{MRC} = \frac{\left( \sum_{n=0}^{N-1} \frac{z_n}{\sigma_n^2} z_n \right)^2}{\sum_{n=0}^{N-1} \frac{z_n^2}{\sigma_n^2}} = \sum_{n=0}^{N-1} \gamma_n. \tag{2.38} \]

By assuming that \( R_n \)'s are identical, i.e., \( R = R_0 \approx \cdots \approx R_n \approx \cdots \approx R_{N-1} \), if \( A_1 \)'s are Nakagami distributed according to \((A_{1n}, m_{1n}, \Omega_{1n})\), the sum of \( x^2 = A_{1n}^2 \) is also
Nakagami distributed according to \((x, m_T, \Omega_T)\) with [61, (eq. 80, 96)]

\[
m_T = \frac{\left( \sum_{n=0}^{N-1} \frac{\Omega_{1n}}{\mu_{m_1n}} \right)^2}{\sum_{n=0}^{N-1} \left( \frac{\Omega_{1n}}{m_{1n}} \right)},
\]

\[
\Omega_T = \sum_{n=0}^{N-1} \Omega_{1n}.
\]

(2.39)

Therefore, the PDF of \(\gamma^{MRC}\) can be written as

\[
f_{\gamma^{MRC}}(x) = \frac{1}{\Gamma(m_T)} \left( \frac{m_T}{R\Omega_T} \right)^{m_T} x^{m_T-1} \exp \left( -\frac{m_T x}{R\Omega_T} \right).
\]

(2.40)

2.7.3 Equal Gain Combining

The technique of Equal Gain Combining (EGC) is analogous to the MRC with the branch SNIR equally weighted. This implies that the individual strength of the branch signal is not taken into consideration. The advantage is the simplicity in practical application compared to MRC.

Following a similar procedure as in MRC, at the output of the EGC combiner, the decision variable is

\[
z^{EGC} = \sum_{n=0}^{N-1} z_n.
\]

(2.41)

This results in the SNIR at the output of the combiner,

\[
\gamma^{EGC} = \left( \frac{\sum_{n=0}^{N-1} z_{n1}}{\sum_{n=0}^{N-1} \sigma_n^2} \right)^2.
\]

(2.42)

Because of the difficulty in obtaining the distribution of the sum of random envelope of non-identical diversity branches [55], only the identical and independent (iid) cases are
studied. Thus, the above equation can be written as

\[
\gamma_{\text{EGC}} = \frac{R}{N} \left( \sum_{n=0}^{N-1} A_{1n} \right)^2.
\]  

(2.43)

Since \( A_{1n} \) is Nakagami distributed according to \((A_{1n}, m, \Omega)\), the sum of amplitudes \( x = \Sigma A_{1n} \) is also Nakagami distributed approximately according to \([x, Nm, N^2 (1 - \frac{1}{5m}) \Omega]\) [61, eq.84]. Thus, the PDF of \( \gamma_{\text{EGC}} \) then becomes

\[
f_{\gamma_{\text{EGC}}}(x) = \frac{x^{Nm-1}}{\Gamma(Nm)} \left[ \frac{m}{(1 - \frac{1}{5m}) \Omega R} \right]^N \exp \left[ -\frac{mx}{(1 - \frac{1}{5m}) \Omega R} \right].
\]  

(2.44)

2.8 Summary

In this chapter, the basic properties of UWB signals were outlined, starting with the definition given by the FCC, as well as the limits of the power output and spectrum of UWB pulses. Several signal shapes namely – rectangular pulses, Gaussian pulses, Gaussian monocycle pulses and Gaussian doublet pulses – were presented. The frequency spectrum and signal expression of each pulse were derived and explained. In addition, pulse modulation methods and multiple access techniques for UWB radios were presented. SS techniques were briefly discussed.

The methods for the analysis of UWB wireless communications were presented, starting with the generalized system model. Some characteristics of a propagation channel were considered, such as the number of multipath components, multipath amplitude fading distribution and multipath intensity profiles. Finally, UWB system performance parameters and diversity combining techniques were introduced.
CHAPTER III

TIME HOPPING UWB MULTIPLE ACCESS TECHNIQUES

3.1 Introduction

In this chapter, one of the proposed multiple access schemes, Time Hopping (TH) multiple access as it relates to UWB communication, is examined with various binary pulse modulation techniques. The basic concept of using TH is to control the time of transmitting pulses. Multiple access capability could be achieved by assigning different transmission time slots for each user. Previous work that involved TH multiple access in radar systems is extended by providing a general analytical framework in which the UWB system performance can be evaluated in dense multipath fading and multiple user environments. The numerical results are obtained by employing different modulation techniques, diversity schemes, pulse shapes and channel fading characteristics.

The heart of any UWB system is the pulse generator, which is used for both transmitting and receiving UWB signals. UWB transmitters convert data bits directly to short duration pulses, while receivers must generate a template pulse that matches the incoming waveform [64].

Even though the conventional modulation scheme by modifying a carrier wave is not employed in UWB transmitters, pulses should be modified in order to attach information. Several pulse modulation techniques could be applied in TH UWB systems, such as Pulse
Position Modulation (PPM), Pulse Amplitude Modulation (PAM) and Pulse Shift Keying (PSK).

3.2 Time Hopping-Pulse Position Modulation

In this section, the analysis of the Time Hopping-Pulse Position Modulation (TH-PPM) ultra wideband communication systems is presented. A step-by-step approach is used.

3.2.1 Transmitted TH-PPM Ultra Wideband Signals

By employing PPM and TH techniques, the TH-PPM scheme is designed to achieve multiuser capacity in any UWB system. PPM can be applied to both analog and digital modulation [14]. The key motivation for using this scheme is its relatively low complexity and the ability to highly resolve the multipath signal.

In TH-PPM ultra wideband systems, the typical modulation functions from (2.10) are given by

\[ F(j, k) = 1; \]
\[ G(j, k) = jT_f + c_{jk}T_c + \beta d_{jk}. \]  

Thus, the transmitted TH-PPM symbol signal of the \( k \)th user can be written as

\[ s_k(t) = \sqrt{\frac{E_s}{N_s}} \sum_{j=0}^{N_s-1} p_{tr}(t-t_k-jT_f-c_{jk}T_c-\beta d_{jk}), \]  

where \( N_s \) is the number of pulses per symbol; \( T_f \) and \( T_c \) are the time duration of a frame and a slot respectively; \( c_{jk} \) and \( d_{jk} \) are the time hopping PN sequence and the binary data stream conveying information of the \( k \)th signal; and \( \beta \) is the time shift due to the use of
PPM. This implies that the symbol rate becomes $R_b = \frac{1}{N_s T_f}$ and the symbol duration is $T_b = N_s T_f$.

Obviously, the TH-PPM ultra wideband scheme is accomplished through the time shift of pulses, e.g., the $j$th pulse starts at time $t_k + j T_f + c_{jk} T_c + \beta d_{jk}$. The scheme of time shifting involves the following [14]:

1. $p_{tr}(t - jT_f)$ with uniform pulse train spacing:

   The frame time can be ten to a thousand times the pulse width, resulting in a signal with a very low duty cycle. A frame can be divided into time slots and the pulse generated from the $k$th user occupies only one of these slots. The rest of the slots can be allocated to other users. Due to equally spaced pulses in multiple-user environments, a collision may occur when signals from two users are received simultaneously. This problem can be resolved by the next step of time shifting.

2. Random time hopping

   A distinct PN code set $\{c_{jk}\}$ is signed to each user so that the actual transmission time of each pulse can be shifted. These hopping sequences are periodic codes and each element is an integer within the range $[0, N_h - 1]$. Thus, the TH sequence adds an additional time shift to each monocycle in the pulse train.

   Since a short time interval may be required to process the output pulses, it is assumed that $N_h T_c \leq T_f$. Moreover, the parameter $N_h$ indicates the fraction of the frame time over which TH is allowed [14]. This implies that if the ratio $\frac{N_h T_c}{T_f}$ is too small, the collision may still occur, and if the ratio is large enough with well-designed PN codes, increased multiple access capacity can be achieved. We assume in this case that optimum value of $N_h$ is used
to get the best performance such that the self-interference between pulses inside a symbol could be eliminated, or at least mitigated.

(3) Modulation

Since it is assumed that the modulation data symbol changes every $N_s$ hops, the value of $d_{j1}$ is either 1 or 0 during one symbol transmission. The time shift of $\beta$ is added to transmit a bit 1.

Figure 3.1 shows an example of the shared channel by using TH-PPM rectangular pulses. User 1 transmits a bit 0 by the PN code $\{2, 3, 1, \cdots, 2\}$, such that the pulse takes the second slot in the first frame, the third slot in the second frame, and so on. User 2 follows the same step with the PN code $\{1, 2, 3, \cdots, 1\}$.

![Figure 3.1: Shared channel by using TH-PPM systems.](image)

3.2.2 Received TH-PPM Ultra Wideband Signals

Through a multipath fading channel, the received signal is the sum of signals from $K$ active users and $L$ multipath components, and is given by

$$r(t) = \sqrt{\frac{E_s}{N_s}} \sum_{k=1}^{K} \sum_{l=0}^{L-1} \sum_{j=0}^{N_s-1} A_{kl}p_{rec}(t - t_k - lT_p - jT_f - c_{jk}T_c - \beta d_{jk}) + n(t). \quad (3.3)$$
Due to the use of PPM, the reference signal at the receiver is the difference of bit 0 and bit 1, expressed as

\[ s^*_1(t) = \sqrt{E_p} \sum_{i=0}^{N_s-1} p_{rec}(t - iT_f - c_i T_c) - p_{rec}(t - iT_f - c_1 T_c - \beta), \tag{3.4} \]

where \( E_p \) is chosen to normalize the symbol energy to one, such as \( \int_{-\infty}^{\infty} [s^*_1(t)]^2 = 1 \). Thus, we have

\[ N_s E_p \theta_4 = 1, \tag{3.5} \]

with

\[ \theta_4 = \int_{-\infty}^{\infty} [p_{rec}(t) - p_{rec}(t - \beta)]^2 dt. \tag{3.6} \]

### 3.2.3 Variance of TH-PPM Ultra Wideband Systems

In a TH-PPM ultra wideband system, at the \( n \)th branch, the decision statistic \( z_n \) consist of four terms as shown in (2.21) and is given by

\[
\begin{align*}
z_{n1} &= A_{1n} \sqrt{\frac{E_s}{N_s}} \sum_{j=0}^{N_s-1} R_{p_{rec}^* p_{rec}}(0)
\end{align*}
\]

\[
\begin{align*}
z_{n2} &= \sqrt{\frac{E_s}{N_s}} \sum_{l=0, l \neq n}^{L-1} \sum_{j=0}^{N_s-1} \sum_{i=0}^{N_s-1} A_{1l} R_{p_{rec}^* p_{rec}}(nT_p - lT_p)
\end{align*}
\]

\[
\begin{align*}
z_{n3} &= \sqrt{\frac{E_s}{N_s}} \sum_{k=2}^{K} \sum_{l=0}^{L-1} \sum_{j=0}^{N_s-1} \sum_{i=0}^{N_s-1} A_{kl} R_{p_{rec}^* p_{rec}}(nT_p - lT_p)
\end{align*}
\]

\[
\begin{align*}
z_{n4} &= \sum_{i=0}^{N_s-1} R_{p_{rec}}(nT_p),
\end{align*}
\]

(3.7)

where \( P_{rec}^{p_{rec}}(t) \) and \( P_{rec}^{p_{rec}}(t) \) stand for

\[ P_{rec}^{p_{rec}}(t) = p_{rec}(t - t_k - jT_f - c_j k T_c - \beta d_{jk}), \]
Thus, the general cross-correlation function becomes

\[
P_{j_1}^* (t) = \sqrt{E_p} \left[ p_{rec} (t - iT_f - c_{i_1} T_c) - p_{rec} (t - iT_f - c_{i_1} T_c - \beta) \right]. \quad (3.8)
\]

Thus, the general cross-correlation function becomes,

\[
R_{P_{j_1}^* P_{i_1}^*} (nT_p - lT_p) = \int_0^{T_b} \bar{P}_{j_1} (t + nT_p - lT_p) P_{i_1}^* (t) \, dt,
\]

\[
= \sqrt{E_p} \int_0^{T_b} p_{rec} (t - t_k - jT_f - c_{jk} T_c - \beta d_{jk} + nT_p - lT_p) \times [p_{rec} (t - iT_f - c_{i_1} T_c) - p_{rec} (t - iT_f - c_{i_1} T_c - \beta)] \, dt
\]

\[
= \sqrt{E_p} \int_0^{T_b} p_{rec} (t + \Delta) [p_{rec} (t) - p_{rec} (t - \beta)] \, dt, \quad (3.9)
\]

where \( \Delta = (i - j)T_f + (n - l)T_p + (c_{i_1} - c_{jk})T_c - \beta d_{jk} - t_k \). It is assumed that the time difference \( t_k \) is an RV, and may span many frame time durations, such that

\[
t_k = \begin{cases} 
0, & k = 1, \\
J_k T_f + \varepsilon_k, & k \geq 2,
\end{cases} \quad (3.10)
\]

where \( J_k \) is an integer rounding \( t_k \) to the nearest frame, and \( \varepsilon_k \) is the round error with \(-\frac{T_f}{2} \leq \varepsilon_k \leq \frac{T_f}{2}\). We also assume that \( \varepsilon_k \) is uniformly distributed over \([-\frac{T_f}{2}, \frac{T_f}{2}]\). Thus, the parameter \( \Delta \) becomes

\[
\Delta = (i - j - J_k)T_f + (n - l)T_p + (c_{i_1} - c_{jk})T_c - \varepsilon_k - \beta d_{jk}. \quad (3.11)
\]

For the ideal case, the magnitude of \( \Delta \) is always less than \( T_f \) [14]. Since \( R_{P_{j_1}^* P_{i_1}^*} (nT_p - lT_p) \) is nonzero only for \( |\Delta| < T_p \), the nonzero values may occur when \( i = j + J_k \).

At the \( n \)th branch, the desired signal part is from the \( l \)th multipath component. A binary system has \( \Delta = -\beta d_{jk} \), resulting in

\[
R_{P_{j_1}^* P_{j_1}^*} (0) = \sqrt{E_p} \theta_1, \quad (3.12)
\]

37
\[
\begin{align*}
(\theta_1|d_j = 0) &= \int_0^{T_b} p_{\text{rec}}(t) \left[ p_{\text{rec}}(t) - p_{\text{rec}}(t - \beta) \right] dt, \\
(\theta_1|d_j = 1) &= \int_0^{T_b} p_{\text{rec}}(t - \beta) \left[ p_{\text{rec}}(t) - p_{\text{rec}}(t - \beta) \right] dt. 
\end{align*}
\] (3.13)

Thus, the value of \( z_{n_1}^2 \) becomes
\[
z_{n_1}^2 = \left[ A_1n \sqrt{\frac{E_s}{N_0}} \sum_{j=0}^{N_s-1} R_{PP_{\text{rec}P}^*_{j1}}(0) \right]^2 = A_1 n E_s N_0 E_p \theta_1^2. \tag{3.14}
\]

Due to the extremely short pulses and large spaces between pulses, the self-interference might be only from adjacent multipath components. In a binary system, if a bit 0 is received, the self-interference is nonzero only when \( l = n + 1 \). In other words, at the 1st branch \((n = 0)\) the desired signal is from the 1st multipath component \((l = 0)\) while self-interference is due to the 2nd arrival multipath component \((l = 1)\); at the 2nd branch \((n = 1)\) the self-interference is due to the 3rd arrival multipath component \((l = 2)\), and so on. At the \(N\)th branch, self-interference may not occur. On the other hand, if a bit 1 is received, this value is nonzero only when \( l = n - 1 \). In this case, at the 1st branch, no self-interference could be considered. In general, the cross-correlation function for the self-interference part is written as
\[
R_{PP_{\text{rec}P}^*_{j1}}[(n-l)T_p - d_{j1}\beta] = \sqrt{E_p} \theta_2, \tag{3.15}
\]

with
\[
\begin{align*}
(\theta_2|d_j = 0) &= \int_0^{T_b} p_{\text{rec}}(t - T_p) \left[ p_{\text{rec}}(t) - p_{\text{rec}}(t - \beta) \right] dt, \\
(\theta_2|d_j = 1) &= \int_0^{T_b} p_{\text{rec}}(t + T_p - \beta) \left[ p_{\text{rec}}(t) - p_{\text{rec}}(t - \beta) \right] dt. \tag{3.16}
\end{align*}
\]
This results in the variance of \( z_{n2} \),

\[
\sigma^2_{(z_{n2}|d_j1)} = \text{var}(z_{n2}|d_j1 = 0) = \begin{cases} 
E_s E_p \theta_2^2 \Omega_1(n+1), & 0 \leq n < L - 1, \\
0, & n = L - 1,
\end{cases}
\]  

(3.17)

and

\[
\sigma^2_{(z_{n2}|d_j1)} = \text{var}(z_{n2}|d_j1 = 1) = \begin{cases} 
E_s E_p \theta_2^2 \Omega_1(n-1), & 0 < n \leq L - 1, \\
0, & n = 0.
\end{cases}
\]  

(3.18)

An analysis of the MAI effect uses \( \Delta = (n-l)T_p + (c_{(j+J)b} - c_{jk})T_e - \varepsilon_k - d_{jk}\beta \), so that the variance of the cross-correlation function may be written as

\[
\sigma^2_{R_{p_{rec}p_{t1}^*}} = \text{var} \left[ R_{p_{rec}p_{t1}^*}(nT_p - lT_p) \right] = \frac{E_p}{T_f} \int_{-w}^{w} \int_0^{T_b} \left\{ \int_0^{T_b} [p_{rec}(t) - p_{rec}(t - \beta)] p_{rec} (t + \Delta) dt \right\}^2 d\varepsilon_k
\]

\[= E_p \theta_3, \]

(3.19)

with

\[
\theta_3 = \frac{1}{T_f} \int_{-w}^{w} \int_0^{T_b} \left\{ \int_0^{T_b} p_{rec}(t - \varepsilon_k) [p_{rec}(t) - p_{rec}(t - \beta)] dt \right\}^2 d\varepsilon_k,
\]

(3.20)

where the range of the above integral, \( \pm w = \pm \frac{T_f}{2} + (n-l)T_p + (c_{(j+J)b} - c_{jk})T_e - d_{jk}\beta \) is independent of the sequence variables and the symbol sequence of other \( K - 1 \) users. Thus, the variance of \( z_{n3} \) can be written as

\[
\sigma^2_{z_{n3}} = \text{var}(z_{n3}) = E_s E_p \theta_3 \sum_{k=2}^{K} \sum_{l=0}^{L-1} \Omega_{kl}.
\]

(3.21)
By substituting (3.14), (3.17), (3.18), (3.21) and (A.5) into (2.25), the parameter $R_n$ under the condition $d_{jk}$ becomes

$$R_n|_{d_{jk}} = \frac{E_s N_s R_{prec,\theta_{i1j1}}^2 (0)}{\sigma_n^2}$$

$$= \frac{E_s N_s E_p \theta_1^2}{E_s E_p \theta_2 \Omega_1(d_{jk},n) + E_s E_p \theta_3 \sum_{k=2}^{K} \sum_{l=0}^{L-1} \Omega_{kl} + \frac{N_0}{2}}$$

$$= \frac{\theta_1^2 \Omega_1(d_{jk},n) + \theta_3 \sum_{k=2}^{K} \sum_{l=0}^{L-1} \Omega_{kl} + \frac{1}{2E_s/N_0 \sigma_n^2}}{\theta_1^2 \Omega_1(d_{jk},n) + \theta_3 \sum_{k=2}^{K} \sum_{l=0}^{L-1} \Omega_{kl} + \frac{1}{2E_s/N_0 \sigma_n^2}}. \quad (3.22)$$

If a bit 0 is transmitted, $R_n$ can be written as

$$R_n^0 = \begin{cases} \left\{ \frac{\theta_2 \Omega_1(n+1)}{\sigma_1^2 N_s} + \frac{\theta_3}{\sigma_1^2 N_s} \sum_{k=2}^{K} \sum_{l=0}^{L-1} \Omega_{kl} + \frac{\theta_4}{2\sigma_2^2 E_s/N_0} \right\}^{-1} & 0 \leq n < L - 1, \\ \left\{ \frac{\theta_2}{\sigma_1^2 N_s} \sum_{k=2}^{K} \sum_{l=0}^{L-1} \Omega_{kl} + \frac{\theta_4}{2\sigma_2^2 E_s/N_0} \right\}^{-1} & n = L - 1. \end{cases} \quad (3.23)$$

If a bit 1 is transmitted, $R_n$ becomes

$$R_n^1 = \begin{cases} \left\{ \frac{\theta_2 \Omega_1(n-1)}{\sigma_1^2 N_s} + \frac{\theta_3}{\sigma_1^2 N_s} \sum_{k=2}^{K} \sum_{l=0}^{L-1} \Omega_{kl} + \frac{\theta_4}{2\sigma_2^2 E_s/N_0} \right\}^{-1} & 0 < n \leq L - 1, \\ \left\{ \frac{\theta_2}{\sigma_1^2 N_s} \sum_{k=2}^{K} \sum_{l=0}^{L-1} \Omega_{kl} + \frac{\theta_4}{2\sigma_2^2 E_s/N_0} \right\}^{-1} & n = 0. \end{cases} \quad (3.24)$$

Thus, the variances and autocorrelation functions have been derived.

3.2.4 Pulse Signal Properties

Due to the extremely short pulse duration of UWB signals, the pulse shape may have impacts on the system performance. In order to evaluate the effect of pulse shapes, two types of waveforms are adopted in this study:

(A) Rectangular Waveform

As mentioned earlier, rectangular-shaped pulses are commonly used for system analysis due to their simplicity. In this dissertation, it is assumed that the transmitted pulse and the received pulse are rectangular-shaped, with a unit amplitude in the interval $[0, T_p]$. 40
(B) Gaussian Waveform

In this case, at the input of the transmitter antenna, the signal pulse is Gaussian shape and is given by

\[ p_{tr}(t) = t \exp \left[-\frac{2\pi \left(t - \frac{T_p}{2}\right)^2}{t_m^2}\right]. \quad (3.25) \]

Since the antenna and channel modify the shape of waveforms, the received signal \( p_{rec}(\cdot) \) is modeled as the doublet Gaussian shape, given by [14]

\[ p_{rec}(t) = \left[1 - 4\pi \frac{(t - \frac{T_p}{2})^2}{t_m^2}\right] \exp \left[-\frac{2\pi \left(t - \frac{T_p}{2}\right)^2}{t_m^2}\right]. \quad (3.26) \]

The summary of pulse parameters is given in Table 3.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>System 1</th>
<th>System 2</th>
<th>System 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data transmission rate, ( R_b )</td>
<td>1 Mbps</td>
<td>11 Mbps</td>
<td>33 Mbps</td>
</tr>
<tr>
<td>Number of pulses per symbol, ( N_s )</td>
<td>22</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Frame time, ( T_f )</td>
<td>40 ns</td>
<td>22 ns</td>
<td>14.9 ns</td>
</tr>
<tr>
<td>Pulse width, ( T_p )</td>
<td>0.7ns</td>
<td>0.7ns</td>
<td>0.7ns</td>
</tr>
<tr>
<td>PPM delay, ( \beta )</td>
<td>0.156ns</td>
<td>0.156ns</td>
<td>0.156ns</td>
</tr>
<tr>
<td>Doublet Gaussian pulses, ( \theta^1_1 )</td>
<td>0.0305</td>
<td>0.0305</td>
<td>0.0305</td>
</tr>
<tr>
<td>Doublet Gaussian pulses, ( \theta^2_2 )</td>
<td>3.1358\times10^{-8}</td>
<td>3.1358\times10^{-8}</td>
<td>3.1358\times10^{-8}</td>
</tr>
<tr>
<td>Doublet Gaussian pulses, ( \theta^3_3 )</td>
<td>1.5121\times10^{-4}</td>
<td>2.7476\times10^{-4}</td>
<td>4.0585\times10^{-4}</td>
</tr>
<tr>
<td>Doublet Gaussian pulses, ( \theta^4_4 )</td>
<td>0.3492</td>
<td>0.3492</td>
<td>0.3492</td>
</tr>
<tr>
<td>Rectangular pulses, ( \theta^1_1 )</td>
<td>0.0243</td>
<td>0.0243</td>
<td>0.0243</td>
</tr>
<tr>
<td>Rectangular pulses, ( \theta^2_2 )</td>
<td>0.0243</td>
<td>0.0243</td>
<td>0.0243</td>
</tr>
<tr>
<td>Rectangular pulses, ( \theta^3_3 )</td>
<td>3.7838\times10^{-4}</td>
<td>1.3759\times10^{-3}</td>
<td>2.3015\times10^{-3}</td>
</tr>
<tr>
<td>Rectangular pulses, ( \theta^4_4 )</td>
<td>0.3120</td>
<td>0.3120</td>
<td>0.3120</td>
</tr>
</tbody>
</table>

3.2.5 Signal-to-Noise plus Interference Ratio for TH-PPM Systems

The evaluation of the SNIR performance is based on the assumption that the signal pulse width is much smaller than the frame duration and the arrivals of all resolvable multipath components are within one frame. The identical Nakagami fading parameter \( m \) for
each propagation path is also assumed. Therefore, with equal probability of transmitting bit 0 and bit1, the average SNIR at the output of combiner is given by

$$\gamma = \frac{1}{2} \int_0^\infty x f_\gamma(x|\text{bit 0}) dx + \frac{1}{2} \int_0^\infty x f_\gamma(x|\text{bit 1}) dx, \quad (3.27)$$

where $f_\gamma(x)$ is the PDF of the instantaneous SNIR at the output of the diversity combiner, defined in (2.35), (2.40) and (2.44), conditioned on the bits, for SD, MRC and EGC, respectively.

Particularly for an identical integer value of the Nakagami $m$-parameter, the conditional average SNIR at the output of SD can be written as

$$\gamma^{SD}_{\text{bit } d} = \int_0^\infty x f_{\gamma^{SD}}(x|\text{bit } 0) dx$$

$$= \int_0^\infty \sum_{n=0}^{N-1} \left( \frac{m}{\Omega_1 R_n^d} \right)^m \frac{x^m}{\Gamma(m)} \exp \left( -\frac{m x}{\Omega_1 R_n^d} \right)$$

$$\times \prod_{i=0, i\neq n}^{N-1} \frac{G \left( m, \frac{m x}{\Omega_i R_i^d} \right)}{\Gamma(m)} dx \quad (3.28)$$

Let $y = \frac{m x}{\Omega_1 R_n^d}$, it is obtained that

$$x = \frac{\Omega_1 R_n^d y}{m},$$

$$dx = \frac{\Omega_1 R_n^d}{m} dy. \quad (3.29)$$

For any positive integers, the gamma function has $\Gamma(m) = (m-1)!$ so that (3.28) becomes

$$\gamma^{SD}_{\text{bit } d} = \sum_{n=0}^{N-1} \frac{\Omega_1 R_n^d}{\Gamma(m+1)} \int_0^\infty \exp (-y) y^m$$

$$\times \prod_{i=0, i\neq n}^{N-1} \left[ 1 - \exp \left( -\frac{\Omega_1 R_n^d y}{\Omega_i R_i^d} \right) \sum_{a=0}^{m-1} \left( \frac{\Omega_1 R_n^d y}{\Omega_i R_i^d} \right)^a a! \right] dy, \quad (3.30)$$
where $\Omega_{1n}$ is defined in (2.29); and the parameter $R^d_n$ is different for transmitting bit 0 ($d = 0$) and bit 1 ($d = 1$), as described in (3.23) and (3.24). The above equation could be calculated by using 16-point Gauss-Laguerre integration [65], [66], as shown in Appendix B.

With the MRC technique, the mean value of $R^d_n$ (noted as $R^d$) is used in order to simplify the analysis. Thus, the average SNIR at the output of MRC is given by

$$
\gamma^{MRC} = \frac{1}{2} \int_0^\infty x f_{\gamma^{MRC}}(x|\text{bit 0}) dx + \frac{1}{2} \int_0^\infty x f_{\gamma^{MRC}}(x|\text{bit 1}) dx
$$

$$
= \frac{1}{2} \int_0^\infty \frac{1}{\Gamma(m_T)} \left( \frac{m_T}{R^0 \Omega_T} \right)^{m_T} x^{m_T} \exp \left( -\frac{m_T x}{R^0 \Omega_T} \right) dx
$$

$$
+ \frac{1}{2} \int_0^\infty \frac{1}{\Gamma(m_T)} \left( \frac{m_T}{R^1 \Omega_T} \right)^{m_T} x^{m_T} \exp \left( -\frac{m_T x}{R^1 \Omega_T} \right) dx,
$$

(3.31)

where $R^0$ and $R^1$ are the mean values conditioned in bit 1 and 0 respectively; and $m_T$ and $\Omega_T$ are defined in equation (2.39). Let $y = \frac{m_T x}{R^0 \Omega_T}$, therefore

$$
x = \frac{R^0 \Omega_T y}{m_T},
$$

$$
dx = \frac{R^0 \Omega_T}{m_T} dy.
$$

(3.32)

Thus, (3.31) becomes

$$
\gamma^{MRC} = \frac{R^0 \Omega_T}{2\Gamma(m_T + 1)} \int_0^\infty y^{m_T} \exp (-y) dy
$$

$$
+ \frac{R^1 \Omega_T}{2\Gamma(m_T + 1)} \int_0^\infty y^{m_T} \exp (-y) dy
$$

$$
= \frac{1}{2} (R^0 + R^1) \Omega_T.
$$

(3.33)

Similarly, the average SNIR at the output of EGC can be written as

$$
\gamma^{EGC} = \frac{1}{2} \int_0^\infty x f_{\gamma^{EGC}}(x|\text{bit 0}) dx + \frac{1}{2} \int_0^\infty x f_{\gamma^{EGC}}(x|\text{bit 1}) dx.
$$

(3.34)
By substituting (2.40) into (3.34), the above equation becomes

\[
\gamma_{EGC} = \frac{1}{2} \int_0^\infty \frac{x^{Nm}}{\Gamma(Nm)} \left[ \frac{m}{(1 - \frac{1}{5m}) \Omega R^0} \right]^{Nm} \exp \left[ -\frac{m x}{(1 - \frac{1}{5m}) \Omega R^0} \right] dx
\]

\[
+ \frac{1}{2} \int_0^\infty \frac{x^{Nm}}{\Gamma(Nm)} \left[ \frac{m}{(1 - \frac{1}{5m}) \Omega R^1} \right]^{Nm} \exp \left[ -\frac{m x}{(1 - \frac{1}{5m}) \Omega R^1} \right] dx
\]

\[
= \frac{(1 - \frac{1}{5m}) \Omega R^0}{2 \Gamma(Nm + 1)} \int_0^\infty x^{Nm} \exp(-x) \, dx
\]

\[
+ \frac{(1 - \frac{1}{5m}) \Omega R^1}{2 \Gamma(Nm + 1)} \int_0^\infty x^{Nm} \exp(-x) \, dx
\]

\[
= \frac{1}{2} \left( R^0 + R^1 \right) N \left( 1 - \frac{1}{5m} \right) \Omega.
\]

(3.35)

In Figure 3.2, the SNIR performances of diversity combiners are compared for three

\[\text{Figure 3.2: Average SNIR comparison for TH-PPM systems.}\]
different UWB systems when using the number of active users $K = 15$, the Nakagami fading parameter $m = 1$ and the diversity order $N = 5$. All curves are obtained by using the uniform MIP and the Gaussian waveform. It is noted that in each UWB system, three diversity combining techniques show the same tendency in terms of the average SNIR as increasing $E_s/N_0$. Thus, in order to avoid repeating similar results, only one combiner technique might be chosen for further analysis. Obviously, the use of MRC brings the best SNIR performance while the use of SD shows the lowest SNIR performance. In addition, the average SNIR of EGC is very close to that of MRC.

As expected, the more $E_s/N_0$ is increased, the higher average SNIR is achieved. However, this improvement of performance depends on the system characteristics. For example, in Systems 2 and 3, the increase of the average SNIR is not distinct at higher $E_s/N_0$. It is also noticed that the best SNIR performance could be achieved in System 1, while System 3 presents the worst SNIR performance. This implies that data transmission rates could have negative impacts on the average SNIR.

In Figure 3.3, the trade-off between the data transmission rate and the multiple access capacity is evaluated in three different UWB systems. The plot is obtained by using MRC with the application of Gaussian waveforms and the uniform MIP. For a single-user transmission ($K = 1$), there is no difference of the SNIR performance among the three systems. However, when the number of active users increases, significant differences could be observed from graphical results. For instance, System 1 is not very sensitive to the number of active users, but the average SNIR sharply decreases when the number of active users increases in System 2 and 3. This indicates that in order to maintain the required average SNIR, the data transmission rate might be sacrificed (33 Mbps in System 3, 11 Mbps in
System 2 and 1 Mbps in System 1). Therefore, increasing the diversity order \( N \) to maintain the SNIR performance (\( N \) from 1 to 5 in this example) will satisfy both required transmission bit rates and multiple access capacity.

![Graph showing trade-off between number of active users, \( K \), and data transmission rate in TH-PPM systems](image)

Figure 3.3: Trade-off between the number of active users, \( K \), and data transmission rate in TH-PPM systems \((m = 1)\) and \( E_s/N_0 = 10 \) dB).

The SNIR performance as a function of the diversity order \( N \) is plotted in Figure 3.4 in System 2 with values of \( K = 10 \) and \( E_s/N_0 = 10 \) dB. The diversity order plays an important role in determining the average SNIR. As the number of diversity order increases, the better SNIR performance could be achieved. In addition, the big difference is shown between no diversity \((N = 1)\) and other diversity orders \((N \leq L)\). This improvement becomes smaller at higher \( E_s/N_0 \) or at higher \( N \). This implies that it might not be necessary...
to choose the maximal diversity order $N$ to achieve the certain system performance, in relation to decrease circuit complexity. In addition, when using the SD technique, the change of the average SNIR is not distinct as increasing the diversity order.

![Graph](image)

**Figure 3.4:** Average SNIR as a function of the diversity order, $N$, in a TH-PPM system.

Figure 3.5 shows the average SNIR for Gaussian and rectangular shaped pulses with the assumption of both uniform and exponential MIP. The plot is obtained by assuming $m = 1$, $K = 5$ and $N = 5$ with the application of MRC. There is not much difference between the use of uniform MIP and exponential MIP in the three UWB systems, especially when $E_s/N_0$ is less than 5 dB in this figure. Therefore, one MIP distribution may be used for the future discussion to avoid redundant description. Obviously, the use of Gaussian
pulses shows the better SNIR performance than the use of rectangular pulses in Systems 2 and 3. This is evidence of the importance of the pulse shape design. It is also noted that rectangular pulses might be used as substitutes of Gaussian pulses when analyzing the performance of UWB System 1.

![Graph showing the effect of TH-PPM pulse shapes on the average SNIR performance.](image)

**Figure 3.5:** Effect of TH-PPM pulse shapes on the average SNIR performance.

### 3.2.6 Bit Error Rate for TH-PPM Systems

In this section, the analytical expressions that can be used to evaluate the BER performance are derived. It is assumed that diversity branches are independent and the interferences plus noise are Gaussian distributed. This implies that the conditional BER at the
output of the diversity combiner can be written as

\[ P_b(\gamma|\text{bit } d) = Q\left(\sqrt{\gamma^d}\right), \quad (3.36) \]

where \( d \in \{0, 1\} \) in a binary system and \( Q(\cdot) \) is the Q-function, which can be equivalently written as [62, 3.44]

\[ Q(\gamma) = \frac{1}{(1 - \frac{1}{\pi}) \gamma + \frac{1}{\pi} \sqrt{\gamma^2 + 2\pi} \sqrt{2\pi} \exp\left(-\frac{\gamma^2}{2}\right)}. \quad (3.37) \]

Hence, the average BER at the output of the diversity combiner can be written as

\[ \overline{P}_b = \frac{1}{2} \int_0^\infty P_b(\gamma)f_\gamma(x|\text{bit } 0)dx + \frac{1}{2} \int_0^\infty P_b(\gamma)f_\gamma(x|\text{bit } 1)dx, \quad (3.38) \]

where the PDF of the SNIR, \( f_\gamma(x) \) is defined in (2.35), (2.40) and (2.44) for SD, MRC and EGC, respectively.

By assuming identical integer Nakagami fading parameter \( m \), if SD is applied, the average BER becomes,

\[ \overline{P}_b = \frac{1}{2} \int_0^\infty Q(\sqrt{\gamma})f_{\gamma SD}(x|\text{bit } 0)dx + \frac{1}{2} \int_0^\infty Q(-\sqrt{\gamma})f_{\gamma SD}(x|\text{bit } 1)dx \\
= \frac{1}{2} (I_0 + I_1). \quad (3.39) \]

For \( d \in \{0, 1\} \), the generalized expression becomes

\[ I_d = \int_0^\infty Q(\sqrt{\gamma})f_{\gamma SD}(x|\text{bit } d)dx. \quad (3.40) \]

By substituting (2.35) into (3.40), the above equation is given by

\[ I_d = \sum_{n=0}^{N-1} \int_0^\infty Q(\sqrt{\Omega_1 R_{d_n}^d}) \left(\frac{m}{\Omega_1 R_{d_n}^d}\right)^m \frac{x^{m-1}}{\Gamma(m)} \exp\left(-\frac{mx}{\Omega_1 R_{d_n}^d}\right) \prod_{i=0, i \neq n}^{N-1} \frac{G\left(m, \frac{mx}{\Omega_1 R_{d_i}^d}\right)}{\Gamma(m)} dx \]
\[ I_d = \sum_{n=0}^{N-1} \int_0^\infty \exp (-x) \frac{x^{m-1}}{\Gamma(m)} Q \left( \frac{n R_d x}{\Omega} \right) \times \prod_{i=0,i\neq n}^{N-1} \left[ 1 - \exp \left( \frac{-\Omega R_d x}{\Omega R_i} \right) \sum_{a=0}^{m-1} \frac{\left( \frac{\Omega R_d x}{\Omega R_i} \right)^a}{a!} \right] dx, \]  

(3.41)

where incomplete gamma function \( G(m, x) \) is defined in (2.33). 16-point Gauss-Laguerre integration could be used to solve the above integration [65], [66], as shown in Appendix B.

Similarly, the average BER performance at the output of MRC can be written as

\[
\bar{P}^{MRC}_b = \frac{1}{2} \int_0^\infty Q(\sqrt{\gamma}) f_{\gamma, MRC} (x | \text{bit 0}) dx + \frac{1}{2} \int_0^\infty Q(-\sqrt{\gamma}) f_{\gamma, MRC} (x | \text{bit 1}) dx
\]

\[
= \frac{1}{2} (I_0 + I_1), \quad \quad (3.42)
\]

with

\[
I_0 = \int_0^\infty Q(\sqrt{x}) \left( \frac{m_T}{R^0 \Omega_T} \right)^{m_T} \frac{x^{m_T-1}}{\Gamma(m_T)} \exp \left( -\frac{m_T x}{R^0 \Omega_T} \right) dx,
\]

\[
I_1 = \int_0^\infty Q(-\sqrt{x}) \left( \frac{m_T}{R^1 \Omega_T} \right)^{m_T} \frac{x^{m_T-1}}{\Gamma(m_T)} \exp \left( -\frac{m_T x}{R^1 \Omega_T} \right) dx, \quad (3.43)
\]

where \( m_T \) and \( \Omega_T \) are defined in equation (2.39); and \( R^d \) is the mean value of \( R^d_n \). Let \( x = y^2 \), it is obtained that

\[
I_0 = \frac{2}{\Gamma(m_T)} \left( \frac{m_T}{R^0 \Omega_T} \right)^{m_T} \int_0^\infty y^{2(m_T-1)-1} \exp \left( -\frac{m_T y^2}{R^0 \Omega_T} \right) Q(y) dy.
\]

\[
I_1 = \frac{2}{\Gamma(m_T)} \left( \frac{m_T}{R^1 \Omega_T} \right)^{m_T} \int_0^\infty y^{2(m_T-1)-1} \exp \left( -\frac{m_T y^2}{R^1 \Omega_T} \right) Q(-y) dy. \quad (3.44)
\]

Since \( Q(y) = Q(-y) = 1 - \Phi(y) \), by using [67, 6.286, 1], the above equations are evaluated to give
\[
I_0 = \frac{2}{\sqrt{\pi(2m_T-1)}} \left( \frac{m_T}{R^0 \Omega_T} \right)^{m_T} 2F_1 \left[ \frac{2m_T-1}{2}; m_T; \frac{2m_T-1}{2}; \left( \frac{m_T}{R^0 \Omega_T} \right)^2 \right],
\]
\[
I_1 = \frac{2}{\sqrt{\pi(2m_T-1)}} \left( \frac{m_T}{R^1 \Omega_T} \right)^{m_T}
\times 2F_1 \left[ \frac{2m_T-1}{2}; m_T; \frac{2m_T-1}{2}; \left( \frac{m_T}{R^1 \Omega_T} \right)^2 \right],
\]
(3.45)

where \(2F_1(a; b; c; x)\) is the hypergeometric function [67].

The average BER at the output of EGC is then given by

\[
P_{b}^{EGC} = \frac{1}{2} \int_{0}^{\infty} Q(\sqrt{\gamma}) f(x \vert \text{bit 0}) dx + \frac{1}{2} \int_{0}^{\infty} Q(-\sqrt{\gamma}) f(x \vert \text{bit 1}) dx
\]
\[
= \frac{1}{2} (I_0 + I_1),
\]
(3.46)

with

\[
I_0 = \int_{0}^{\infty} Q(\sqrt{x}) \frac{x^{Nm-1}}{\Gamma(Nm)} \left[ \frac{m}{(1 - \frac{1}{5m}) \Omega R^0} \right]^{Nm} \exp \left[ - \frac{mx}{(1 - \frac{1}{5m}) \Omega R^0} \right] dx,
\]
\[
I_1 = \int_{0}^{\infty} Q(-\sqrt{x}) \frac{x^{Nm-1}}{\Gamma(Nm)} \left[ \frac{m}{(1 - \frac{1}{5m}) \Omega R^1} \right]^{Nm} \times \exp \left[ - \frac{mx}{(1 - \frac{1}{5m}) \Omega R^1} \right] dx.
\]
(3.47)

Similarly, the closed form of the above equation could be given by

\[
I_0 = \left[ \frac{m}{(1 - \frac{1}{5m}) \Omega R^0} \right]^{Nm}
\times 2F_1 \left\{ \frac{2Nm-1}{2}; Nm; \frac{2Nm-1}{2}; \left[ \frac{m}{(1 - \frac{1}{5m}) \Omega R^0} \right]^2 \right\},
\]
\[
I_1 = \frac{2}{\sqrt{\pi(2Nm-1)}} \left[ \frac{m}{(1 - \frac{1}{5m}) \Omega R^0} \right]^{Nm}
\times 2F_1 \left\{ \frac{2Nm-1}{2}; Nm; \frac{2m_T-1}{2}; \left[ \frac{m_T}{(1 - \frac{1}{5m}) \Omega R^0} \right]^2 \right\}.
\]
(3.48)
In Figure 3.6, the BER is compared for three UWB systems by employing MRC, EGC and SD. The comparison is under the condition of Gaussian-shaped pulses, uniform MIP, active users $K = 5$, RAKE branches $N = 5$ and the Nakagami parameter $m = 2$.

From Figure 3.6, it is observed that the MRC technique shows the best performance in terms of the BER, while the EGC technique takes second place. Only small differences could be seen between the MRC and EGC techniques. The effect of increasing $E_s/N_0$ on the average BER is different in three systems. For instance, in System 1, the bit error rate keeps sharply decreasing as $E_s/N_0$ increases; in System 2, the tardiness of this decrease
could be seen; in System 3, only small effect is shown from the figure. Therefore, it can be concluded that System 1 has the best performance among three UWB systems, but with the lowest transmission bit rate (1 Mbps). Contrarily, System 3 shows the highest transmission bit rate (33 Mbps) but with the worst performance in terms of the BER.

In Figure 3.7, the average BER is evaluated as a function of the number of active users for three proposed UWB systems at $m = 2$, $N = 5$ and $E_s/N_0 = 10$ dB. If no multiple access capacity is needed (i.e., $K = 1$), the three systems show the same BER performance as expected. As the number of active users increases, the bit error rate increases, and the system performance decreases, such that when there are 15 active users by employing MRC,
the BER goes up to 0.01%, 1.1% and 6.6% in Systems 1, 2 and 3 respectively. Obviously, the trade-off between the BER and data transmission rate is the major concern in UWB systems. As before, the EGC diversity technique shows the very close BER performance to the MRC.

In order to achieve the better system performance while maintaining the required data transmission rate, the BER is evaluated as the function of the diversity order, the fading parameter and $E_s/N_0$, as shown in Figures 3.8 and 3.9. Figure 3.8 is obtained with $K = 5$, $m = 1$ and $E_s/N_0 = 8$ dB for Gaussian pulses over an independent and identically distributed (iid) fading channel in an EGC ultra wideband system. The diversity order plays
a significant role in determining the system performance. System 1 always shows the lowest bit error rate (best performance). When the diversity order increases, the gaps among three systems are bigger. One phenomenon should be mentioned, such as, when the diversity order $N$ is greater than 8, little increase could be observed in SD ultra wideband systems. In this case, increasing the diversity order does not improve the system performance as it does in EGC ultra wideband systems.

In Figure 3.9, the effect of the channel fading on the BER is evaluated in UWB Systems 1 and 2. The plot is obtained by using $K = 5$ and $N = 5$ under the assumption of Gaussian pulses over an iid channel. It is noticed that the performance improvement is not

![Figure 3.9: Average BER of TH-PPM as a function of the fading parameter, $m$.](image_url)

In Figure 3.9, the effect of the channel fading on the BER is evaluated in UWB Systems 1 and 2. The plot is obtained by using $K = 5$ and $N = 5$ under the assumption of Gaussian pulses over an iid channel. It is noticed that the performance improvement is not
distinct as the fading decreases ($m$ increases) at lower $E_s/N_0$. It can also be observed that there exists a significant difference between $m = 1$ and other Nakagami fading parameters. For example in System 1, at the error rate of $10^{-3}$, the required $E_s/N_0$ is about 9.2 dB, 7.1 dB, 6.6 dB, 6.3 dB and 6.1 dB for $m = 1, 2, 3, 4,$ and 5, respectively.

In Figure 3.10, the BER in SD ultra wideband systems is compared for two types of waveforms and two types of MIPs when $K = 5$, $N = 5$ and $m = 1$. The results obtained from this figure are similar to the ones from Figure 3.5. It is evident that the transmitted signal power has very important impacts on the performance in terms of the BER. In System 1, no significant difference could be seen between the use of Gaussian
pulse and rectangular pulses. However, in System 2, when using the rectangular pulses for the analysis, the BER deteriorates. Since no significant difference is shown between the uniform MIP and the exponential MIP, using the uniform MIP for the analysis could avoid any redundant description.

3.2.7 Outage Probability for TH-PPM Systems

Based on the definition of the outage probability in (2.29), with the given threshold \( \varepsilon \), the outage probability becomes

\[
P_{\text{out}} = \frac{1}{2} \int_0^\varepsilon f_\gamma(x \mid \text{bit 0}) dx + \frac{1}{2} \int_0^\varepsilon f_\gamma(x \mid \text{bit 1}) dx,
\]

where \( f_\gamma(x) \) is the PDF of the SNIR for the corresponding diversity combining technique, defined in (2.35), (2.40) and (2.44).

If SD is applied, the outage probability becomes

\[
P_{\text{out}}^{\text{SD}} = \frac{1}{2} \int_0^\varepsilon f_\gamma^{\text{SD}}(x \mid \text{bit 0}) dx + \frac{1}{2} \int_0^\varepsilon f_\gamma^{\text{SD}}(x \mid \text{bit 1}) dx
\]

\[
= \frac{1}{2} (I_0 + I_1),
\]

with

\[
I_d = \sum_{n=0}^{N-1} \int_0^\infty \left( \frac{m}{\Omega_{1n} R_n^0} \right)^m x^{m-1} \frac{\Gamma(m)}{\Gamma(m)} \exp \left( - \frac{mx}{\Omega_{1n} R_n^0} \right)
\]

\[
\times \prod_{i=0, i \neq n}^{N-1} \frac{G \left( m, \frac{mx}{\Omega_{1i} R_i^0} \right)}{\Gamma(m)} dx
\]

\[
= \frac{1}{\Gamma(m)} \sum_{n=0}^{N-1} \int_0^{\Omega_{1n} R_n^0} x^{m-1} \exp (-x)
\]

\[
\times \prod_{i=0, i \neq n}^{N-1} \left[ 1 - \exp \left( - \frac{\Omega_{1n} R_n^0 x}{\Omega_{1i} R_i^0} \right) \sum_{a=0}^{m-1} \frac{\left( \frac{\Omega_{1n} R_n^0 x}{\Omega_{1i} R_i^0} \right)^a}{a!} \right] dx.
\]
where \( a \) is an integer. Gaussian numerical integration could be used to calculate the above equation (66), as presented in Appendix B.

When using the MRC scheme, the outage probability is given by

\[
P_{\text{out}}^{\text{MRC}} = \frac{1}{2} \int_0^\varepsilon f_{\gamma,\text{MRC}}(x|\text{bit } 0)dx + \frac{1}{2} \int_0^\varepsilon f_{\gamma,\text{MRC}}(x|\text{bit } 1)dx
\]

\[
= \frac{1}{2} \int_0^\varepsilon \frac{1}{\Gamma(m_T)} \left( \frac{m_T}{R^0\Omega_T} \right)^{m_T} x^{m_T-1} \exp \left( -\frac{m_T x}{R^0\Omega_T} \right) dx + \frac{1}{2} \int_0^\varepsilon \frac{1}{\Gamma(m_T)} \left( \frac{m_T}{R^1\Omega_T} \right)^{m_T} x^{m_T-1} \exp \left( -\frac{m_T x}{R^1\Omega_T} \right) dx
\]

\[
= \frac{1}{2\Gamma(m_T)} (I_0 + I_1),
\]

(3.52)

with

\[
I_0 = \int_0^{\varepsilon m_T} x^{m_T-1} \exp (-x) dx,
\]

\[
I_1 = \int_0^{\varepsilon m_T} x^{m_T-1} \exp (-x) dx,
\]

(3.53)

where the definitions of \( m_T \) and \( \Omega_T \) could be found in equation (2.39). By using [67, 3.381, 2], the above equations become

\[
I_0 = \exp \left( -\frac{\varepsilon m_T}{R^0\Omega_T} \right) \sum_{a=0}^\infty (-1)^a \frac{\left( \varepsilon m_T \right)^{a+m_T}}{m_T (m_T + 1) \cdots (m_T + a)}
\]

\[
I_1 = \exp \left( -\frac{\varepsilon m_T}{R^1\Omega_T} \right) \sum_{a=0}^\infty (-1)^a \frac{\left( \varepsilon m_T \right)^{a+m_T}}{m_T (m_T + 1) \cdots (m_T + a)}.
\]

(3.54)

Similarly, if EGC is applied, the outage probability can be written as

\[
P_{\text{out}}^{\text{EGC}} = \frac{1}{2} \int_0^\varepsilon f_{\gamma,\text{EGC}}(x|\text{bit } 0)dx + \frac{1}{2} \int_0^\varepsilon f_{\gamma,\text{EGC}}(x|\text{bit } 1)dx
\]

58
\[ P_{\text{out}}^{\text{EGC}} = \frac{1}{2} \int_0^\varepsilon x^{Nm-1} \left[ \frac{m}{1 - \frac{1}{5m}} \Omega R^0 \right]^{Nm} \exp \left[ -\frac{mx}{(1 - \frac{1}{5m}) \Omega R^0} \right] dx \]

\[ + \frac{1}{2} \int_0^\varepsilon x^{Nm-1} \left[ \frac{m}{1 - \frac{1}{5m}} \Omega R^1 \right]^{Nm} \exp \left[ -\frac{mx}{(1 - \frac{1}{5m}) \Omega R^1} \right] dx \]

\[ = \frac{1}{2\Gamma(Nm)} (I_0 + I_1), \quad (3.55) \]

with

\[ I_0 = \exp \left[ -\frac{\varepsilon m}{(1 - \frac{1}{5m}) \Omega R^0} \right] \sum_{a=0}^\infty (-1)^a \frac{(\varepsilon m)^a}{Nm (Nm+1) \cdots (Nm+a)} \]

\[ I_1 = \exp \left[ -\frac{\varepsilon m}{(1 - \frac{1}{5m}) \Omega R^1} \right] \sum_{a=0}^\infty (-1)^a \frac{(\varepsilon m)^a}{Nm (Nm+1) \cdots (Nm+a)}. \quad (3.56) \]

Numerical results for the outage probability performance are given in Figures 3.11 and 3.12 by assuming \( m = 2 \) and \( E_s/N_0 = 10 \) dB and using Gaussian pulses and the uniform MIP. In Figure 3.11, the outage probability is plotted as a function of the threshold \( \varepsilon \) and the number of active users, \( K \) in MRC ultra wideband systems when \( N = 3 \). As always, there is no difference among the three systems when there is only one active user in each system. If 5 users transmit UWB Gaussian pulses simultaneously, to achieve an outage probability of \( 10^{-3} \), the threshold \( \varepsilon \) could be approximately chosen as 4.3 dB, 1.8 dB and -1.2 dB for Systems 1, 2 and 3 respectively. It is also evident that at a given threshold \( \varepsilon \), System 3 shows the highest outage probability while System 1 shows the lowest. Hence, the best performance is achieved in System 1.

In Figure 3.12, we investigate the outage probability versus the diversity characteristics in System 1. Such characteristics include the multipath channel condition and types of diversity combiners. All curves are obtained by assuming \( K = 12 \). As expected, the outage
Figure 3.11: Outage probability of TH-PPM as a function of the number of active users, $K$. 
probability decreases as the diversity order increases. That again implies the best system performance could be obtained by adding more diversity branches in MRC ultra wideband systems. If there is only one diversity branch, the three diversity combining schemes show the same outage probability.

![Outage Probability Graph](image)

**Figure 3.12:** Outage probability of TH-PPM as a function of the diversity order, $N$.

3.3  Time Hopping-Phase Shift Keying

3.3.1  Transmitted TH-PSK Ultra Wideband Signals

As indicated above, UWB pulses could also be modulated by using PSK. In this case, the TH-PSK modulation functions (2.10) are given by
\[
F(j, k) = 2d_{jk} - 1, \\
G(j, k) = jT_f + c_{jk}T_c,
\]

(3.57)

where \(d_{jk} \in \{0, 1\}\) is a user bit modulated by PSK, in which a bit 1 is transmitted by using pulses at zero degrees and a bit 0 is transmitted by using pulses at 180 degrees; \(T_f\) and \(T_c\) are the frame time duration and the slot time duration respectively; and \(c_{jk}\) is the TH hopping sequence with \(c_{jk} \in \{0, 1, \ldots, N_h\}\). The parameter \(N_h\) has the same meaning as that in TH-PPM ultra wideband systems, standing for the number of time hopping slots. Hence, the transmitted TH-PSK signal symbol of the \(k\)th user becomes

\[
s_k(t) = \sqrt{E_s N_s} \sum_{j=0}^{N_s-1} (2d_{jk} - 1) p_{tr} (t - t_k - jT_f - c_{jk}T_c). \tag{3.58}
\]

In this case, the data transmission rate and the symbol duration are the same with those in TH-PPM ultra wideband systems.

Similar to TH-PPM ultra wideband systems, the multiple access capacity is achieved by using the TH scheme in TH-PSK systems. However, the pulse modulation is based on modifying pulse phases. Figure 3.13 shows the same example of a shared channel by using the PSK technique. User 1 transmits a bit 0 with the PN code \(\{2, 3, 1, \cdots, 2\}\), and user 2 transmit a bit 1 with the PN code \(\{1, 2, 3, \cdots, 1\}\).

3.3.2 Received TH-PSK Ultra Wideband Signals

By employing PSK, the received signal can be written as

\[
r(t) = \sqrt{E_s N_s} \sum_{k=1}^{K} \sum_{l=0}^{L-1} \sum_{j=0}^{N_s-1} A_{kl} (1 - 2d_{jk}) p_{rec} (t - t_k - jT_f - c_{jk}T_c - lT_p) + n(t). \tag{3.59}
\]
Figure 3.13: Shared channel by using TH-PSK systems.

Assuming that the receiver has perfect knowledge of the channel characteristics and the desired signal is from user 1, the template used in the pulse correlator is given by

\[
s_1^*(t) = \sum_{i=0}^{N_s-1} p_{\text{rec}}(t - iT_f - c_1T_c),
\]

where \( E_p \) is the pulse energy with \( \int_{0}^{\infty} [s_1^*(t)]^2 \, dt = 1 \). Thus, we have

\[
N_s E_p \theta_1 = 1,
\]

with

\[
\theta_1 = \int_0^{T_b} p_{\text{rec}}^2(t) \, dt.
\]

### 3.3.3 Variance of TH-PSK Systems

In a TH-PSK ultra wideband system, the expression of \( z_n \) is the same with (3.7), but with different notations of \( P_{jk}^{\text{rec}}(t) \) and \( P_{i1}^*(t) \), given by

\[
P_{jk}^{\text{rec}}(t) = (1 - 2d_{jk}) p_{\text{rec}}(t - t_k - jT_f - c_{jk}T_c),
\]

\[
P_{i1}^*(t) = \sqrt{E_p} p_{\text{rec}}(t - iT_f - c_{i1}T_c).
\]
Thus, the general cross-correlation function becomes,

\[
R_{P_{jk}P_{i1}}(nT_p - lT_p) = \sqrt{E_p} \left(1 - 2d_{jk}\right) \int_0^{T_b} p_{rec}(t - iT_f - c_{i1}T_c) \times p_{rec}(t - t_k - jT_f - c_{jk}T_c + nT_p - lT_p) dt
\]

\[
= \sqrt{E_p} \left(1 - 2d_{jk}\right) \int_0^{T_b} p_{rec}(t + \Delta)p_{rec}(t)dt, \quad (3.64)
\]

where \(\Delta = (i - j)T_f + (n - l)T_p + (c_{i1} - c_{jk})T_c - t_k\). The transmission time difference \(t_k\) has the same properties as shown in (3.10) and (3.11). Thus, the above equation is nonzero when and only when \(|\Delta| < T_p\).

By using a method similar to that used in evaluating TH-PPM ultra wideband systems, it can be shown that the signal power is given by

\[
z_{n1}^2 = A_{1n}^2 E_s N_s E_p \theta_1^2. \quad (3.65)
\]

It is worth pointing out that there is no self-interference from multipath components in TH-PSK ultra wideband systems if there is enough space between pulses. Due to the MAI effect, the variance of \(z_{n3}\) can be written as

\[
\sigma_{z_{n3}}^2 = \text{var} (z_{n3}) = E_s E_p \sigma_3^2 \sum_{k=2}^{K} \sum_{l=0}^{L-1} \Omega_{kl}, \quad (3.66)
\]

with

\[
\sigma_3^2 = \frac{1}{T_f} \int_{-w}^{w} \left[ \int_0^{T_b} p_{rec}(t - \epsilon_k)p_{rec}(t) dt \right]^2 d\epsilon_k, \quad (3.67)
\]

where the range of the above integral, \(\pm w = \pm \frac{T_f}{2} + (n-l)T_p + (c_{(j+J_k)} - c_{jk})T_c\) is independent of the sequence variables and the symbol sequence of other \(K - 1\) users. Therefore, the
value of \( R_n \) is the same at each diversity branch, written as

\[
R = \frac{E_s N_s R_{j_1}^2 (0)}{\sigma_n^2} = \frac{E_s N_s R_{j_1}^2 (0)}{E_s E_p \theta_3^2 \sum_{k=2}^{K} \sum_{l=0}^{L-1} \Omega_{kl} + \frac{N_0}{2}} \]

\[
= \left( \frac{\theta_1^2 \sum_{k=2}^{K} \sum_{l=0}^{L-1} \Omega_{kl} + \frac{1}{2\theta_1 E_s/N_0}}{\theta_3^2 N_s} \right)^{-1} . \tag{3.68}
\]

Due to the use of PSK, some modulation parameters are different with those in TH-PPM ultra wideband systems, as shown in Table 3.2. Only Gaussian pulse shapes are used for the numerical results.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>System 1</th>
<th>System 2</th>
<th>System 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmission Rate, ( R_b )</td>
<td>1 Mbps</td>
<td>11 Mbps</td>
<td>33 Mbps</td>
</tr>
<tr>
<td>Number of pulses per symbol, ( N_s )</td>
<td>22</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Frame time, ( T_f )</td>
<td>40 ns</td>
<td>22 ns</td>
<td>14.9 ns</td>
</tr>
<tr>
<td>Maximum number of active users, ( K )</td>
<td>13</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Pulse width, ( T_p )</td>
<td>0.7 ns</td>
<td>0.7 ns</td>
<td>0.7 ns</td>
</tr>
<tr>
<td>Doublet Gaussian pulses, ( \theta_1 )</td>
<td>0.1079</td>
<td>0.1079</td>
<td>0.1079</td>
</tr>
<tr>
<td>Doublet Gaussian pulses, ( \theta_3 )</td>
<td>4.3136\times10^{-5}</td>
<td>7.8237\times10^{-5}</td>
<td>1.1584\times10^{-4}</td>
</tr>
</tbody>
</table>

### 3.3.4 Signal-to-Noise plus Interference Ratio for TH-PSK Systems

Under the assumption that the signal pulse width is much smaller than the frame duration and the arrivals of all resolvable multipath components are within one time frame, the average SNIR at the output of combiner is given by

\[
\bar{\gamma} = \int_0^\infty x f_\gamma(x) dx. \tag{3.69}
\]

When the respective PDF are substituted in (3.68), it can be shown that with the identical Nakagami \( m \)-parameter, the average SNIR at the output of the selection diversity
combiner becomes
\[
\gamma^{SD} = \int_0^\infty x f_{\gamma^{SD}}(x)\,dx \\
= \int_0^\infty \sum_{n=0}^{N-1} \left( \frac{m}{\Omega_{1n} R} \right)^m x^m \exp \left( -\frac{m x}{\Omega_{1n} R} \right) \prod_{i=0, i\neq n}^{N-1} \frac{G\left(m, \frac{m x}{\Omega_{1i} R}\right)}{\Gamma(m)} \,dx \\
= \frac{R}{\Gamma(m+1)} \sum_{n=0}^{N-1} \Omega_{1n} \int_0^\infty \exp (-x) x^m \prod_{i=0, i\neq n}^{N-1} \left[ 1 - \exp \left( -\frac{\Omega_{1n} x}{\Omega_{1i}} \right) \sum_{a=0}^{m-1} \frac{\left( \frac{\Omega_{1n} x}{\Omega_{1i}} \right)^a}{a!} \right] \,dx. 
\] (3.70)

If the MRC technique is applied at the receiver, the average SNIR can be written as
\[
\gamma^{MRC} = \int_0^\infty x f_{\gamma^{MRC}}(x)\,dx \\
= \int_0^\infty \frac{1}{\Gamma(m_T)} \left( \frac{m_T}{R \Omega_T} \right)^{m_T} x^{m_T} \exp \left( -\frac{m_T x}{R \Omega_T} \right) \,dx \\
= \frac{R \Omega_T}{\Gamma(m_T + 1)} \int_0^\infty y^{m_T} \exp (-y) \,dy \\
= R \Omega_T, 
\] (3.71)

where \( \Omega_T \) is defined in (2.39).

When using EGC, the average SNIR becomes
\[
\gamma^{EGC} = \int_0^\infty x f_{\gamma^{EGC}}(x)\,dx \\
= \int_0^\infty \frac{x^{Nm}}{\Gamma(Nm)} \left[ 1 - \frac{m}{1 - \frac{1}{sm}} \right]^{Nm} \exp \left[ -\frac{m x}{\left( 1 - \frac{1}{sm} \right) \Omega R} \right] \,dx \\
= \frac{R N \left( 1 - \frac{1}{sm} \right) \Omega}{\Gamma(Nm + 1)} \int_0^\infty y^{Nm} \exp (-y) \,dy \\
= R N \left( 1 - \frac{1}{5m} \right) \Omega. 
\] (3.72)

Figure 3.14 shows the effect of the MIP distribution on the average SNIR for TH-PSK systems by employing MRC. The average SNIR is plotted by assuming \( m = 1, K = 4 \).
and $N = 5$. At lower $E_s/N_0$, there is little difference between the use of uniform and exponential MIP. At higher $E_s/N_0$ (between 10 and 20 dB), the gap between two MIPs becomes even smaller in System 1. Even though the difference between two MIPs becomes distinct as increasing $E_s/N_0$ in Systems 2 and 3, the curves still show the same tendency. Therefore, we will use the uniform MIP for the further analysis of TH-PSK ultra wideband systems.

Figure 3.14: Effect of pulse shapes on the average SNIR in TH-PSK systems.

In Figure 3.15, the SNIR comparison of the three different systems is presented for $K = 10$, $m = 1$ and $N = 5$. Like the results shown in TH-PPM systems, the use of MRC brings the best SNIR performance while the use of SD shows the lowest SNIR performance.
Furthermore, System 1 shows the best SNIR performance and System 3 shows the worst SNIR performance under the same comparison conditions.

![SNIR comparison for TH-PSK systems.](image)

Figure 3.15: SNIR comparison for TH-PSK systems.

Figures 3.16 and 3.17 show the comparison of PPM and PSK schemes in TH UWB systems with values of $m = 1$, $N = 3$ and $E_s/N_0 = 10$ dB. It is evident that under the same condition PSK always shows the higher SNIR than PPM. In order to achieve the better SNIR performance without losing multiple access capacity, we can either decrease the data transmission rate or increase the diversity order. However, the data rate is critical in current wireless communications, so the optimal receiver design becomes more important.
to improve the system performance, including choosing the proper modulation technique and diversity scheme.

![SNIR comparison of PSK and PPM as a function of the number of active users and data transmission rate.](image)

Figure 3.16: SNIR comparison of PSK and PPM as a function of the number of active users and data transmission rate.

### 3.3.5 Bit Error Rate for TH-PSK Systems

With assumptions that the diversity branches are independent and the interferences plus noise are Gaussian distributed, the conditional BER at the output of the diversity combiner can be written as

\[ P_b(\gamma) = Q(\sqrt{\gamma}). \]  

(3.73)
Figure 3.17: SNIR Comparison of PSK and PPM, as a function of $N$ and $E_s/N_0$. 
By assuming integer Nakagami fading parameter $m$, the average BER at the output of a SD diversity combiner can be written as

$$P_{SD}^{b} = \int_{0}^{\infty} Q(\sqrt{x}) f_{\gamma_{SD}}(x) \, dx$$

$$= \sum_{n=0}^{N-1} \int_{0}^{\infty} Q(\sqrt{x}) \left( \frac{m}{\Omega_{1n} R} \right)^{m} x^{m-1} \exp \left( -\frac{mx}{\Omega_{1n} R} \right) \Gamma(m) \prod_{i=0, i\neq n}^{N-1} G \left( m, \frac{m_{i}}{\Omega_{1i} R} \right) \, dx$$

$$= \frac{1}{\Gamma(m)} \sum_{n=0}^{N-1} \int_{0}^{\infty} \exp(-x) x^{m-1} Q \left( \sqrt{\frac{\Omega_{1n} R x}{m}} \right) \, dx\prod_{i=0, i\neq n}^{N-1} \left[ 1 - \exp \left( -\frac{\Omega_{1n} x}{\Omega_{1i}} \right) \sum_{a=0}^{m-1} \left( \frac{\Omega_{1n} x}{\Omega_{1i}} \right)^{a} \frac{a!}{a!} \right] \, dx. \quad (3.74)$$

When MRC is applied at the receiver, by using [67, 6.286,1], the average BER becomes

$$P_{MRC}^{b} = \int_{0}^{\infty} Q(\sqrt{x}) f_{\gamma_{MRC}}(x) \, dx$$

$$= \int_{0}^{\infty} Q(\sqrt{x}) \left( \frac{m_{T}}{R\Omega_{T}} \right)^{m_{T}} x^{m_{T}-1} \exp \left( -\frac{m_{T} x}{R\Omega_{T}} \right) \, dx$$

$$= \frac{2 \left( \frac{m_{T}}{R\Omega_{T}} \right)^{m_{T}}}{\Gamma(m_{T})} \int_{0}^{\infty} \left[ 1 - \Phi (y) \right] y^{(2m_{T}-1)-1} \exp \left( -\frac{m_{T} y^{2}}{R\Omega_{T}} \right) \, dy$$

$$= \frac{2 \left( \frac{m_{T}}{R\Omega_{T}} \right)^{m_{T}}}{\sqrt{\pi} (2m_{T} - 1)} {\text{2F1}} \left[ \begin{array}{c} \frac{2m_{T} - 1}{2}; m_{T}; \frac{2m_{T} - 1}{2}; \left( \frac{m_{T}}{R\Omega_{T}} \right)^{2} \end{array} \right], \quad (3.75)$$

where $m_{T}$ and $\Omega_{T}$ are defined in equation (2.39) and $\text{2F1} (a; b; c; x)$ is the hypergeometric function.

Similarly, if EGC is applied, the average BER is given by

$$P_{EGC}^{b} = \int_{0}^{\infty} Q(\sqrt{x}) f_{\gamma_{EGC}}(x) \, dx$$

$$= \int_{0}^{\infty} Q(\sqrt{x}) \frac{x^{Nm-1}}{\Gamma(Nm)} \left[ \frac{m}{(1 - \frac{1}{5m}) \Omega R} \right]^{Nm} \exp \left[ -\frac{m x}{(1 - \frac{1}{5m}) \Omega R} \right] \, dx$$
\[ T_b^{EGC} = 2 \left[ \frac{m}{(1 - \frac{1}{3m}) \Omega R} \right]^{Nm} \int_0^\infty Q(y) y^{(2Nm-1)-1} \exp \left[ -\frac{my^2}{(1 - \frac{1}{3m}) \Omega R} \right] dy \]

\[ = 2 \left[ \frac{m}{(1 - \frac{1}{3m}) \Omega R} \right]^{Nm} \frac{1}{\sqrt{\pi} (2Nm - 1)} \times _2F_1 \left\{ \frac{2Nm - 1}{2}; Nm; \frac{2Nm - 1}{2}; \left[ \frac{m}{(1 - \frac{1}{3m}) \Omega R} \right]^2 \right\}. \] (3.76)

The comparison of the BER for the three UWB systems for MRC, EGC and SD is illustrated in Figure 3.18, when \( K = 8 \), \( N = 5 \) and \( m = 2 \). The same conclusions could be drawn from this plot. System 1 shows the best performance in terms of the BER, but the
lowest transmission bit rate; and using diversity combiners (MRC and EGC) can achieve a lower bit error rate.

TH-PSK and TH-PPM ultra wideband systems have similar results in terms of the BER. However, TH-PSK ultra wideband systems always show better system performance than TH-PPM ultra wideband systems. It is evident that System 1 has the higher multiuser capacity by using both pulse modulation techniques. However, the bit rate is sacrificed (1 Mbps), as shown in Figure 3.19. Furthermore, when adding more diversity branches to both systems, the difference between PSK and PPM is more distinct, as shown in Figure 3.20.

![Figure 3.19: BER comparison of PSK and PPM, as a function of $K$ ($m = 2$, $N = 5$ and $E_s/N_0 = 10$ dB).](image-url)
Figure 3.20: BER comparison of PSK and PPM as a function of $N$ ($m = 2$, $K = 8$ and $E_s/N_0 = 10$ dB).
3.3.6 Outage Probability for TH-PSK Systems

With the given threshold $\varepsilon$, the outage probability of a TH-PSK ultra wideband system can be written as

$$P_{\text{out}} = \int_0^{\varepsilon} f_\gamma(x) dx. \quad (3.76)$$

By substituting $f_\gamma(x)$ in (2.35), (2.40) and (2.44) for each of the diversity combining technique respective, the outage probability could be achieved.

In particular, at the output of SD, the outage probability becomes

$$P_{\text{out}}^{SD} = \int_0^{\varepsilon} f_{\gamma,SD}(x) dx$$

$$= \frac{1}{\Gamma(m)} \sum_{n=0}^{N-1} \int_0^{\varepsilon} \left( \frac{m}{\Omega_{1n} R} \right)^m \frac{x^{m-1}}{\Gamma(m)} \exp \left( -\frac{mx}{\Omega_{1n} R} \right) \prod_{i=0, i \neq n}^{N-1} \frac{G \left( m, \frac{mx}{\Omega_{1i} R} \right)}{\Gamma(m)} dx$$

$$= 1 \Gamma(m) \sum_{n=0}^{N-1} \int_0^{\varepsilon} x^{m-1} \exp (-x) \prod_{i=0, i \neq n}^{N-1} \left[ 1 - \exp \left( -\frac{\Omega_{1n} x}{\Omega_{1i}} \right) \sum_{a=0}^{m-1} \left( \frac{\Omega_{1i} x}{\Omega_{1i}} \right)^a \right] dx. \quad (3.78)$$

When using the MRC scheme, the outage probability can be obtained by using [67, 3.381, 2], given by

$$P_{\text{out}}^{MRC} = \int_0^{\varepsilon} f_{\gamma,MRC}(x) dx$$

$$= \int_0^{\varepsilon} \left( \frac{m_T}{R\Omega_T} \right)^{m_T} \frac{x^{m_T-1}}{\Gamma(m_T)} \exp \left( -\frac{m_T x}{R\Omega_T} \right) dx$$

$$= \frac{1}{\Gamma(m_T)} \int_0^{\varepsilon m_T} x^{m_T-1} \exp (-x) dx,$$

$$= \exp \left( \frac{-\varepsilon m_T}{R\Omega_T} \right) \sum_{a=0}^{\infty} (-1)^a \frac{\left( \frac{\varepsilon m_T}{R\Omega_T} \right)^{a+m_T}}{m_T (m_T + 1) \cdots (m_T + a)}, \quad (3.79)$$

where $m_T$ and $\Omega_T$ are defined in equation (2.39).
Similarly, when EGC is applied, the outage probability becomes

\[
P_{\text{out}}^{\text{EGC}} = \int_0^\varepsilon f_{\gamma_{\text{EGC}}}(x)dx
\]

\[
= \int_0^\varepsilon \frac{x^{Nm-1}}{\Gamma(Nm)} \left[ \frac{m}{(1 - \frac{1}{5m}) \Omega R} \right]^{Nm} \exp \left[ - \frac{mx}{(1 - \frac{1}{5m}) \Omega R} \right] dx
\]

\[
= \frac{1}{\Gamma(Nm)} \int_0^{\varepsilon \left( \frac{m}{\Omega R} \right)^{-\frac{1}{5m}} x^{Nm-1}} \exp (-x) dx
\]

\[
= \exp \left[ - \frac{\varepsilon m}{(1 - \frac{1}{5m}) \Omega R} \right] \sum_{a=0}^{\infty} (-1)^a \frac{\left( \frac{\varepsilon m}{\Omega R} \right)^{a+Nm}}{Nm(Nm+1) \cdots (Nm+a)} \tag{3.80}
\]

The outage probability performance of a TH-PSK system is shown in Figures 3.21 and 3.22 for the constant \( m = 1 \) and \( E_s/N_0 = 10 \) dB. In Figure 3.21, the comparison of the outage probability in several UWB systems is plotted by using \( K = 5 \) and \( N = 3 \). At a given threshold \( \varepsilon = 2 \) dB, the probability of failing to achieve a minimum SNIR of 2 dB is as small as \( 10^{-6} \) in System 1 with the application of MRC, but becomes as high as 95.61% in System 3 when using SD.

Reaching the lower outage probability in System 3 with the application of SD can be achieved by adding more diversity branches or decreasing the number of active users, as shown in Figure 3.22. Decreasing the number of active users from 5 to 1 even without using multipath diversity (i.e., \( N = 1 \)), decreases the outage probability to 8.01%. In addition, if the diversity order is changed to 6, the outage probability is close to zero. Obviously, the diversity order plays a significant role in improving the system performance in such systems.
Figure 3.21: Comparison of the outage probability in TH-PSK systems.
Figure 3.22: Outage probability as a function of $\varepsilon$, $K$ and $N$ in TH-PSK systems.
3.4 Time Hopping-Pulse Amplitude Modulation

By employing Pulse Amplitude Modulation (PAM) and Time Hopping (TH) techniques, the TH-PAM scheme can achieve increased multiuser capacity in a UWB system. However, since the UWB signal power density is very low, it would be difficult to distinguish the change in amplitude, especially in a fading environment. Therefore, when using PAM, pulses are modulated by modifying the sign of amplitudes, for example a positive amplitude (1) for bit 1s and a negative amplitude (-1) for bit 0s. From this point of view, PAM shows the same modulation expressions and properties as PSK.

3.5 Conclusions

In this chapter, the performance analysis of TH ultra wideband systems is presented. It is shown that the use of TH can have several effects on the system performance. The key condition is to distribute the pulses over time to avoid data collisions. Next, TH can reduce the power spectrum density. Thus, the signal can be made virtually indistinguishable from white noise for undesired or unauthorized receivers. Finally, the low complexity of the TH technique makes it an outstanding candidate in UWB communication applications.

The analytical expressions obtained in this chapter can be used to determine the performance of TH UWB systems. These expressions were given for the SNIR, BER and outage performance for different diversity techniques. The following can be concluded from this chapter:

(1) As more users are added to an UWB system, degradation in the system performance is observed. This is consistent with published studies of wireless multiple access systems. To obtain higher multiple access capacity, the data transmission rate is sacrificed.
(2) In order to maintain both data transmission rate and system performance, the diversity techniques could be applied by taking into account the multipath diversity. As the diversity order increases, the performance of the system also increases. However, the performance improvement is not significant at higher diversity orders.

(3) The use of Gaussian pulses improves the system performance compared to rectangular pulses.

(4) Choosing the proper modulation technique is another strategy to improve the system performance. Our results have shown that binary TH-PSK or TH-PAM achieves better performance than conventional TH-PPM.

(5) Like all other wireless communication systems, TH UWB systems may undergo serious channel fading. The increased transmitted power may improve the system performance. However, at higher $E_s/N_0$, the improvement is not distinct.

(6) Comparing the three diversity techniques, it is shown that MRC might be the best candidate in determining system performance, while the use of SD shows the worst system performance. However, one benefit of SD is its simple implementation.
CHAPTER IV

DIRECT SEQUENCE MULTIPLE ACCESS UWB SYSTEM

4.1 Introduction

Direct Sequence Spread Spectrum (DSSS) is a well-known multiple access technique in digital wireless communications, especially in cellular mobile radio communications. When using the DSSS technique, several users are allowed to share the same frequency band without interference from each other. The users’ information is controlled by a Pseudo-random Noise (PN) spreading code sequence. When the data signal is modulated by the PN sequence, a system known as Direct Sequence-Code Division Multiple Access (DS-CDMA) is obtained.

In the early 1990s, DS-CDMA was adapted for civilian wireless systems after several years of use by the military. This technique was standardized as the U. S. interim standard IS-95. Later, it was adopted by Japanese and European standardization bodies for the definition of third-generation wireless systems and it is believed that DS-CDMA is envisioned for UWB systems [60]. Recently, many researchers have proposed this technique to UWB communications [17], [20], [32]-[34]. However, most published research to date has not focused on the application of diversity combiners in proposed UWB systems. In this dissertation, current research results are expanded upon by evaluating the system per-
formance with the application of diversity techniques in a multipath, fading, and multiuser environment.

4.2 DS-PSK Ultra Wideband Signals

The multiple access capability is set up by assigning each user a unique PN spreading sequence out of a family of orthogonal sequences and by having each user operate independently with no knowledge of the other users. In a DS-CDMA system, users employ Binary Phase Shift Keying (BPSK) and Code Division Multiple Access (CDMA) to achieve multiuser capacity. In this dissertation, a system known as DS-PSK ultra wideband is analyzed. The typical signal modulation functions of the \( k \)th transmitting user are given by

\[
F(j, k) = b_{jk}d_{jk}; \\
G(j, k) = jT_p. \tag{4.1}
\]

where \( T_p \) is the duration of a chip, also the pulse width, \( \{b_{jk}\} \) is the PN sequence, and \( d_{jk} \in \{-1, 1\} \) is the binary data stream modulated by BPSK. If a transmitted user’s bit is \( d' \in \{0, 1\} \), then \( d_{jk} = 2d' - 1 \). Thus, the transmitted UWB signal symbol of the \( k \)th user is given by

\[
s_k(t) = \sqrt{\frac{E_s}{N_s}} \sum_{j=0}^{N_s-1} b_{jk}d_{jk}p_{tr} \left( t - t_k - jT_p \right), \tag{4.2}
\]

where \( N_s \) represents the processing gain in DS-PSK ultra wideband systems, e.g., the number of chips per symbol.

Figure 4.1 shows a simple example for a DS-PSK ultra wideband signal format. The plot describes the transmitted signal by using a rectangular waveform with the PN code sequence \( \{b_{jk}\} = \{-1-1 1111 -1-111 -11-11\} \). Each pulse represents a chip, with an am-
plitude equal to $+1$ or $-1$, and has a duration of $T_p$. The data waveform is a time sequence of non-overlapping rectangular pulses, each of which also has an amplitude equal to $+1$ or $-1$. Thus, each symbol has a duration of $T_b = N_s T_p$. When a bit 1 is transmitted, any $d_{jk} = 1$ is mapped to $\{b_{jk}\}$ by BPSK modulation and spreading. Similarly, when a bit 0 is transmitted, any $d_{jk} = -1$ is mapped to $\{-b_{jk}\}$. To achieve the MA capacity, each user has a distinct PN sequence.

![Diagram of signal format in a DS-PSK ultra wideband system.](image)

**Figure 4.1:** Signal format in a DS-PSK ultra wideband system.

Through a multipath fading channel, the received signal is given by

$$r(t) = \sqrt{\frac{E_s}{N_0}} \sum_{k=1}^{K} \sum_{l=0}^{L-1} \sum_{j=0}^{N_s-1} A_{kl} b_{jk} d_{jk} p_{rec} (t - t_k - jT_p - lT_p) + n(t). \quad (4.3)$$
If the signal from user 1 is desired by the receiver, the reference signal can be written as

\[ s_1^*(t) = \sqrt{E_p} \sum_{i=0}^{N_s-1} b_{1i}p_{\text{rec}}(t - iT_p), \]  \tag{4.4}

where \( E_p \) is chosen to obtain that \( \int_0^\infty |s_1^*(t)|^2 dt = 1 \), with

\[ N_s E_p \theta_1 = 1 \]  \tag{4.5}

where \( \theta_1 = \int_{-\infty}^\infty p_{\text{rec}}^2(t) dt \).

4.3 Variance of DS-PSK Ultra Wideband Systems

In DS-PSK ultra wideband systems, the notations of \( P_{jk}^{\text{rec}}(t) \) and \( P_{11}^*(t) \) stand for

\[ P_{jk}^{\text{rec}}(t) = b_{jk}d_{jk}p_{\text{rec}}(t - t_k - jT_p) \]
\[ P_{11}^*(t) = \sqrt{E_p}b_{1i}p_{\text{rec}}(t - iT_p). \]  \tag{4.6}

Thus, the general cross-correlation function can be written as

\[ R_{P_{jk}^{\text{rec}}P_{11}^*}(nT_p - lT_p) = \int_{-\infty}^{\infty} P_{jk}^{\text{rec}}(t + nT_p - lT_p)P_{11}^*(t) dt \]
\[ = \sqrt{E_p}b_{jk}b_{1i}d_{jk} \int_{-\infty}^{\infty} p_{\text{rec}}(t - \Delta) p_{\text{rec}}(t) dt, \]  \tag{4.7}

where \( \Delta = (i - j)T_p + (n - l)T_p - t_k \).

Assuming that the perfect synchronization has been achieved at the receiver, at the \( n \)th branch the desired signal term from the \( l \)th multipath component is given by

\[ z_{n1} = A_{1n} \sqrt{\frac{E_s E_p}{N_s}} \sum_{j=0}^{N_s-1} b_{1j}b_{j1}d_{j1} \int_{-\infty}^{\infty} p_{\text{rec}}^2(t) dt \]
\[ = A_{1n} \sqrt{E_s E_p N_s} (\pm 1) \theta_1, \]  \tag{4.8}

where \( \theta_1 = \int_{-\infty}^{\infty} p_{\text{rec}}^2(t) dt \), with the fact that \( b_{j1}b_{j1} = 1 \) and \( d_{j1} \in \{\pm 1\} \).
The effect of self-interference due to the multipath fading is illustrated in Figure 4.2 by using rectangular pulses. It is assumed that the data bit $d_{j1}$ keeps constant inside a symbol period.

Therefore, at the $n$th diversity branch, the self-interference term can be written as

$$z_{n2} = \sqrt{\frac{E_s E_p}{N_s}} \sum_{l=0, l\neq n}^{L-1} \sum_{j=0}^{N_s-1} \sum_{i=0}^{N_s-1} A_{1l} b_{j1} b_{i1} d_{j1} \int_{-\infty}^{\infty} p_{rec}(t + \Delta) p_{rec}(t) dt$$

$$= \sqrt{\frac{E_s E_p}{N_s}} \sum_{l=0, l\neq n}^{L-1} A_{1l} W_{1l}, \quad (4.9)$$

Figure 4.2: Illustration of self-interference in a DS-PSK system.
for $l > n$, and $W_1$ is

\[
W_1 = b_1 b_{i1} d_{j1} \int_{-\infty}^{\infty} p_{rec}(t + \Delta) p_{rec}(t) dt
\]

\[
= \theta_1 d_1^0 \sum_{i=0}^{l-n-1} b_{(N_s+i+n-l)} b_{i1} + \theta_1 d_1^{-1} \sum_{i=l-n}^{N_s-1} b_{(i+n-l)} b_{i1}
\]

\[
= \theta_1 \left[ X(j) + \tilde{X}(j) \right],
\]

(4.10)

where $d_1^{-1}$ and $d_1^0$ are the consecutive data bits of the $k$th signal, while $d_1^0$ represents the information bit being detected and $d_1^{-1}$ is the preceding bit due to the channel delay, which affects the detection of $d_1^0$ [68, pp. 34], [69]; and $X(j)$ and $\tilde{X}(j)$ are the partial auto correlation functions of the regenerated desired code at the receiver. As seen in Figure 4.2, since the local PN sequence is a periodic code, when $l < n$ the same results could be obtained.

In addition, the random variables of $x = b_1 b_{i1}$ are independent Bernoulli trials and satisfy $Pr(x = 1) = Pr(x = -1) = \frac{1}{2}$. By denoting the cardinality of the set $P$ as $|P|$ and the cardinality of the set $Q$ as $|Q|$, the PDF for $X(j)$ and $\tilde{X}(j)$ are given as [70]

\[
f_{X(j)} = \binom{|P|}{j + |P| - 2} 2^{-|P|}, \; j = -|P|, -|P| + 2, ..., |P| - 2, |P|,
\]

\[
f_{\tilde{X}(j)} = \binom{|Q|}{j + |Q| - 2} 2^{-|Q|}, \; j = -|Q|, -|Q| + 2, ..., |Q| - 2, |Q|,
\]

(4.11)

where $\binom{n}{k}$ represents the binomial coefficient. Since $P$ and $Q$ are mutually independent with $|P| + |Q| = N_s$, it can be shown that

\[
E \left[ X(j)^2 | Q \right] = |P| = N_s - |Q|,
\]

\[
E \left[ \tilde{X}(j)^2 | Q \right] = |Q|.
\]

(4.12)
Therefore, the variance of \( z_{n2} \) is found to be

\[
\sigma_{z_{n2}}^2 = \text{var} \left( z_{n2} \right) \\
= \text{var} \left\{ \sqrt{\frac{E_s E_p}{N_s}} \sum_{l=0, l\neq n}^{L-1} A_{1l}\theta_1 \left[ X(j) + \tilde{X}(j) \right] \right\} \\
= \frac{E_s E_p \theta_1^2}{N_s} (N_s - |Q| + |Q|) \sum_{l=0, l\neq n}^{L-1} \Omega_{1l} \\
= E_s E_p \theta_1^2 \sum_{l=0, l\neq n}^{L-1} \Omega_{1l}. \quad (4.13)
\]

The effect of MAI is illustrated in Figure 4.3. Notice that the time difference \( t_k \) may

![Diagram showing the local PN sequence for user 1 at \( n \)th branch and the effect of MAI.]
span many frame time durations, such that

\[
t_k = \begin{cases} 
0, & k = 1, \\
J'_k T_p + \varepsilon_k, & k \geq 2,
\end{cases} \tag{4.14}
\]

where \(J'_k\) is a nearest integer of \(\frac{t_k}{T_p}\), and \(\varepsilon_k\) is the round error, uniformly distributed over \([-\frac{T_p}{2}, \frac{T_p}{2}]\). Thus, the value of \(\Delta\) in a DS-PSK ultra wideband system becomes

\[
\Delta = (i - j - J'_k)T_p + (n - l)T_p - \varepsilon_k
\]

\[
= (i - j + J_k)T_p - \varepsilon_k, \tag{4.15}
\]

where \(J_k = n - l - J'_k\), as shown in Figure 4.3.

At the \(n\)th diversity branch, the value of \(z_{n3}\) due to the MAI effect can be written as

\[
z_{n3} = \sqrt{\frac{E_s E_p}{N_s}} \sum_{k=2}^{K} \sum_{l=0}^{L-1} \sum_{j=0}^{N_s-1} b_{(i+j+k)} b_{11} b_{1d_j} \int_{-\infty}^{\infty} p_{rec}(t + \Delta) p_{rec}(t) dt
\]

\[
= \sqrt{\frac{E_s E_p}{N_s}} \sum_{k=2}^{K} \sum_{l=0}^{L-1} A_{1W_k}, \tag{4.16}
\]

with

\[
W_k = \left[ d_{k}^{-1} \sum_{i=0}^{J_k-1} b_{(i+J_k+N_s)} b_{11} + d_{k}^{N_s-2} \sum_{j=J_k}^{N_s-1} b_{(i+j+k)} b_{11} + d_{k}^{N_s-2} b_{(N_s-J_k-1)} b_{11} \right] \theta_{31}
\]

\[
+ \left[ d_{k}^{-1} b_{(N_s-J_k-1)} b_{01} + d_{k}^{-1} \sum_{i=0}^{J_k-1} b_{(i+J_k+N_s)} b_{11} \right] \theta_{32}, \tag{4.17}
\]

and

\[
\theta_{31} = \int_{0}^{T_p} p_{rec}(t - \varepsilon_k) p_{rec}(t) dt,
\]

\[
\theta_{32} = \int_{0}^{T_p} p_{rec}(t + T_p - \varepsilon_k) p_{rec}(t) dt. \tag{4.18}
\]
In order to use the method presented in [70], the equation (4.17) can be rearranged to give

\[ W_k = d_k^{-1} \sum_{i=0}^{J_k-1} b_{(i+J_k+N_s)k} \left[ b_{i1} \theta_{31} + b_{(i+1)1} \theta_{32} \right] + d_k^0 \sum_{i=J_k}^{N_s-2} b_{(i+j_k)k} \left[ b_{i1} \theta_{31} + b_{(i+1)1} \theta_{32} \right] + d_k^0 b_{(N_s-J_k-1)k} b_{(N_s-1)1} \theta_{31} + d_k^{-1} b_{(N_s-J_k-1)k} b_{01} \theta_{32}. \]  

(4.19)

To simplify the above equation, a set of RVs \( X_{jk} \) is defined as [60], [70, eq. 12]

\[
X_{ik} = \begin{cases} 
    d_k^{-1} b_{(N_s-J_k-1)k} b_{j1}, & i = 0, \ldots, J_k - 1 \\
    d_k^0 b_{(i+j_k)k} b_{j1}, & i = J_k, \ldots, N_s - 2 \\
    d_k^0 b_{(N_s-J_k-1)k} b_{(N_s-1)1}, & i = N_s - 1 \\
    d_k^{-1} b_{(N_s-J_k-1)k} b_{01}, & i = N_s.
\end{cases}
\]

(4.20)

The RV \( X_{ik} \)'s are mutually independent and satisfy \( \Pr \{ X_{ik} = 1 \} = \Pr \{ X_{ik} = -1 \} = \frac{1}{2} \).

By using the fact that \( b_{i1}^2 = 1 \), (4.19) can be simplified to [60]

\[
W_k = \sum_{i=0}^{N_s-2} X_{ik} \left[ \theta_{31} + b_{i1} b_{(i+1)1} \theta_{32} \right] + X_{(N_s-1)k} \theta_{31} + X_{N_s k} \theta_{32}.
\]

(4.21)

The set \( \{P\} \) may also be defined to the set of integers in the range of \([0, N_s - 2]\) with \(|P|\) possible values of \( i \) such that \( b_{i1} b_{(i+1)1} \), and the set \( \{Q\} \) to the set of integers to the set of integers in the range of \([0, N_s - 2]\) with \(|Q|\) possible values of \( i \) such that \( b_{i1} b_{(i+1)1} = -1 \).

Thus, the above equation becomes

\[
W_k = \sum_{i \in P} X_{ik} \left( \theta_{31} + \theta_{32} \right) + \sum_{i \in Q} X_{ik} \left( \theta_{31} - \theta_{32} \right) + X_{(N_s-1)k} \theta_{31} + X_{N_s k} \theta_{32},
\]

(4.22)

where \( W_k \)'s \((2 \leq k \leq K)\) are mutually independent. The distribution of \( X_{(N_s-1)k} \) and \( X_{N_s k} \) are given by
\[ f_{X_{(N_s-1)k}}(i) = \frac{1}{2}, \quad i = -1, 1, \]
\[ f_{X_{Nsk}}(i) = \frac{1}{2}, \quad i = -1, 1. \] (4.23)

Furthermore, the PDFs of \( \sum_{i \in P} X_{ik} \) and \( \sum_{i \in Q} X_{ik} \) are defined as [60], [70, eq. 20 and 21]

\[ f_{\Sigma_{i \in P} X_{ik}} = \binom{|P|}{i + |P|} 2^{-|P|}, \quad i = -|P|, -|P| + 2, \ldots, |P| - 2, |P|, \]
\[ f_{\Sigma_{i \in Q} X_{ik}} = \binom{|Q|}{i + |Q|} 2^{-|Q|}, \quad i = -|Q|, -|Q| + 2, \ldots, |Q| - 2, |Q|. \] (4.24)

where \( \binom{a}{b} \) represents the binomial coefficient and \( |P| + |Q| = N_s - 1 \). This results in

\[ \text{var} \left( \sum_{i \in P} X_{ik} \right) = |P|, \]
\[ \text{var} \left( \sum_{i \in Q} X_{ik} \right) = |Q| = N_s - 1 - |P|, \] (4.25)

and

\[ \text{var} (X_{(N_s-1)k}) = 1 \]
\[ \text{var} (X_{Nsk}) = 1. \] (4.26)

Since variables \( \sum_{i \in P} X_{ik}, \sum_{i \in Q} X_{ik}, X_{(N_s-1)k}, X_{Nsk}, \theta_{31} \) and \( \theta_{32} \) are maturely independent, by substituting (4.25) into (4.22), the conditional variance of \( W_k \) is given by

\[ \sigma_{W_k|\varepsilon_k}^2 = \text{var} (W_k|\varepsilon_k) \]
\[ = \sum_{|P|=0}^{N_s-2} \frac{1}{N_s - 1} \left[ |P| (\theta_{31} + \theta_{32})^2 + (N_s - 1 - |P|) (\theta_{31} - \theta_{32})^2 + \theta_{31}^2 + \theta_{32}^2 \right] \]
\[ = N_s (\theta_{31}^2 + \theta_{32}^2). \] (4.27)
To take off the condition, the variance of \( W_k \) becomes

\[
\sigma^2_{W_k} = \text{var} (W_k) = \frac{N_s}{T_p} \int_{-\frac{T_p}{2}}^{\frac{T_p}{2}} (\theta^2_{31} + \theta^2_{32}) \, d\varepsilon_k,
\]

which implies that the variance of \( z_{n3} \) is given by

\[
\sigma^2_{z_{n3}} = \text{var} (z_{n3}) = E_s E_p (K - 1) \theta_3 \sum_{l=0}^{L-1} \Omega_{kl},
\]

with

\[
\theta_3 = \frac{1}{T_p} \int_{-\frac{T_p}{2}}^{\frac{T_p}{2}} (\theta^2_{31} + \theta^2_{32}) \, d\varepsilon_k.
\]

By substituting (4.8), (4.13), (4.29) and (A.5) into (2.24), the parameter \( R_n \) becomes

\[
R_n = \frac{E_s N_s E_p \theta^2_1}{E_s E_p \theta_1^2 \sum_{l=0, l \neq n}^{L-1} \Omega_{1l} + E_s E_p (K - 1) \theta_3 \sum_{l=0}^{L-1} \Omega_{kl} + \frac{N_0}{2}}
\]

\[
= \left\{ \frac{1}{N_s} \sum_{l=0, l \neq n}^{L-1} \Omega_{1l} + \frac{\theta_3}{N_s \theta_1^2} \sum_{k=2}^{K} \sum_{l=0}^{L-1} \Omega_{kl} + \frac{1}{2\theta_1 E_s / N_0} \right\}^{-1}.
\]

4.4 System Performance for DS-PSK Ultra Wideband Communications

In order to investigate the performance of DS-PSK UWB systems, the spreading sequences are used based on the Kasami sequence \( (N_s = 15) \) and Gold sequences \( (N_s = 31, N_s = 63, \text{and } N_s = 1023) \) [17], [71]. Other sample parameters used for the numerical results are shown in Table 4.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pulse width, ( T_p )</td>
<td>0.7 ns</td>
</tr>
<tr>
<td>Doublet Gaussian pulses, ( \theta_1 )</td>
<td>0.1079</td>
</tr>
<tr>
<td>Doublet Gaussian pulses, ( \theta_3 )</td>
<td>2.4569 \times 10^{-3}</td>
</tr>
<tr>
<td>Rectangular pulses, ( \theta_1 )</td>
<td>0.7</td>
</tr>
<tr>
<td>Rectangular pulses, ( \theta_3 )</td>
<td>0.5717</td>
</tr>
</tbody>
</table>
4.4.1 Signal-to-Noise plus Interference Ratio

The average SNIR at the output of diversity combiner is defined in (3.69) and the corresponding PDFs, \( f_\gamma (x) \) are defined in (2.35), (2.40) and (2.44) for SD, MRC and EGC respectively. For identical fading parameter \( m \), the average SNIR at the output of SD can be written as

\[
\gamma^{SD} = \int_{-\infty}^{\infty} x f_\gamma^{SD} (x) \, dx \\
= \sum_{n=0}^{N-1} \int_{-\infty}^{\infty} \left( \frac{m}{\Omega_1 R_n} \right)^m x^m \frac{x^m}{\Gamma(m)} \exp \left( - \frac{m x}{\Omega_1 R_n} \right) \prod_{i=0,i\neq n}^{N-1} \frac{G \left( m, \frac{mx}{\Omega_i R_i} \right)}{\Gamma(m)} \, dx \\
= \sum_{n=0}^{N-1} \int_{-\infty}^{\infty} \frac{R_n x^m}{\Gamma(m+1)} \exp (-x) \\
\times \prod_{i=0,i\neq n}^{N-1} \left[ 1 - \exp \left( - \frac{\Omega_1 R_n}{\Omega_1 R_i} x \right) \sum_{a=0}^{m-1} \frac{\left( \Omega_1 R_n / \Omega_1 R_i \right)^a}{a!} \right] \, dx. \tag{4.32}
\]

If the MRC technique is applied at the receiver, the average SNIR can be written as

\[
\gamma^{MRC} = \int_{-\infty}^{\infty} x f_\gamma^{MRC} (x) \, dx \\
= \int_{0}^{\infty} \frac{1}{\Gamma(m_T)} \left( \frac{m_T}{R \Omega_T} \right)^{m_T} x^{m_T} \exp \left( - \frac{m_T x}{R \Omega_T} \right) \, dx \\
= \frac{R \Omega_T}{\Gamma(m_T+1)} \int_{0}^{\infty} y^{m_T} \exp (-y) \, dy \\
= R \Omega_T, \tag{4.33}
\]

where the value \( R \) is the average of \( R_n \)'s, and \( \Omega_T \) is defined in (2.39).

For the EGC, assuming iid channel fading is assumed, it can be shown that average SNIR at the output of EGC is given by
\[ \mathcal{T}^{\text{EGC}} = \int_{-\infty}^{\infty} x f_{\gamma^{E_{G}}} (x) dx \]
\[ = \int_{-\infty}^{\infty} x^{Nm} \left( \frac{m}{(1 - \frac{1}{5m}) \Omega R} \right)^{Nm} \exp \left( -\frac{mx}{(1 - \frac{1}{5m}) \Omega R} \right) dx \]
\[ = \frac{RN \left( 1 - \frac{1}{5m} \right) \Omega}{\Gamma (Nm + 1)} \int_{0}^{\infty} y^{Nm} \exp (-y) dy \]
\[ = \left( 1 - \frac{1}{5m} \right) RN\Omega. \] (4.34)

Figure 4.4 shows the average SNIR as a function of \( E_s/N_0 \) for the three diversity combining techniques. All curves are obtained with the assumption of the uniform MIP, \( N = 5 \), \( m = 1 \), \( K = 10 \) and \( N_s = 63 \). As expected, MRC shows the best SNIR performance, while SD has the worst SNIR performance among the three diversity combining techniques. Un-
expectedly, using rectangular pulses show constantly low average SNIR, so the rectangular waveform might not be applicable for DS-PSK ultra wideband systems. It is also noted that the improvement of the SNIR performance slows down quickly as $E_s/N_0$ increases. This implies that both self-interference and MAI is higher in DS-PSK ultra wideband systems.

Figure 4.5 presents the effect of channel fading and the diversity order on the SNIR performance with $K = 5$, $E_s/N_0 = 10$ dB and $N_s = 63$. Even though the exponential

Figure 4.5: Effect of fading characteristics on the average SNIR in a DS-PSK system.

MIP is considered as the best approach to model the indoor wireless propagations [6], these results do not show a significant difference between the use of uniform MIP and the exponential MIP in evaluating the SNIR performance at decay factor $\mu = 0.1$. Therefore,
only the uniform MIP will be used for the following analysis due to its simplicity. It is also noted that when the fading becomes severe (the Nakagami parameter $m$ decreases), the SNIR performance decreases a little in DS-PSK UWB systems. This implies that DS-PSK systems is not very sensitive the fading propagation environment. As expected, the diversity order $N$ has an important effect on the SNIR performance.

Figure 4.6 compares the average SNIR of DS-PSK ultra wideband systems with different data transmission rates. All curves are obtained by using $E_s/N_0 = 10$ dB and $N = 5$.

![Figure 4.6: Average SNIR of DS-PSK as a function of the number of active users and data transmission rate.](image)

It is noticed that by employing the DS multiple access schemes, the data transmission rate could be very high, such as 95 Mbps in this example. However, at 95 Mbps transmis-
sion rate, with large numbers of active users the average SNIR deteriorates fast. Thus, the transmitted signals might not be recovered by the receiver due to the severe interference.

4.4.2 Bit Error Rate for DS-PSK Ultra Wideband Systems

In order to investigate the Bit Error Rate (BER), it is assumed that the diversity branches are independent and the interferences plus noise are Gaussian distributed. In general, the average BER at the output of the diversity combiner is given by (2.28).

With an integer fading parameter \( m \), the average BER at the output of SD becomes

\[
\overline{P}_b^{SD} = \int_0^\infty Q(\sqrt{x}) f_{\gamma^{SD}}(x)dx
\]

\[
= \int_0^\infty Q(\sqrt{x}) \sum_{n=0}^{N-1} \left( \frac{m}{\Omega_{1n}R_n} \right)^m x^{m-1} \frac{1}{\Gamma(m)} \exp \left( -\frac{m}{\Omega_{1n}R_n} x \right) dx
\]

\[
= \sum_{n=0}^{N-1} \int_0^\infty \exp \left( -x \cdot \frac{m}{\Omega_{1n}R_n} x \right) \frac{1}{\Gamma(m)} \left[ 1 - \exp \left( -\frac{\Omega_{1n}R_n x}{\Omega_{11}R_1} \right) \sum_{a=0}^{m-1} \frac{\Omega_{1n}R_n x}{a!} \right] dx.
\]

Numerical values of above BER could be calculated by 16-point Gauss-Laguerre integration, as shown in Appendix B.

When using MRC, the average BER at the output of MRC can be written as

\[
\overline{P}_b^{MRC} = \int_0^\infty Q(\sqrt{x}) f_{\gamma^{SD}}(x)dx
\]

\[
= \int_0^\infty Q(\sqrt{x}) \frac{1}{\Gamma(m_T)} \left( \frac{m_T}{R_\Omega T} \right)^{m_T} x^{m_T-1} \exp \left( -\frac{m_T x}{R_\Omega T} \right) dx
\]

\[
= 2 \left( \frac{m_T}{R_\Omega T} \right)^{m_T} \frac{1}{\Gamma(m_T)} \int_0^\infty Q(y) y^{(2m_T-1)-1} \exp \left( -\frac{m_T y^2}{R_\Omega T} \right) dy
\]

\[
= 2 \left( \frac{m_T}{R_\Omega T} \right)^{m_T} \frac{1}{\Gamma(m_T)} \int_0^\infty [1 - \Phi(y)] y^{(2m_T-1)-1} \exp \left( -\frac{m_T y^2}{R_\Omega T} \right) dy
\]
by using [67, 6.286, 1], the above equation becomes

\[
P_{b}^{\text{MRC}} = \frac{2 \left( \frac{m_T}{\Omega_T} \right)^{m_T}}{\sqrt{\pi} (2m_T - 1)} 2F_1 \left( \frac{2m_T - 1}{2}; m_T; \frac{2m_T - 1}{2}; \left[ \frac{m_T}{\Omega_T} \right]^2 \right),
\]

where \( m_T \) and \( \Omega_T \) are defined in (2.39).

Similarly, the average BER at the output of EGC is obtained by substituting (3.73) and (2.44) into (2.28), such that

\[
P_{b}^{\text{EGC}} = \int_{0}^{\infty} Q(\sqrt{\gamma}) f_{\gamma,\text{EGC}}(x) dx
\]

\[
= \frac{1}{\Gamma(Nm)} \int_{0}^{\infty} Q(\sqrt{x})x^{Nm-1} \left[ \frac{m}{(1 - \frac{1}{5m}) \Omega R} \right]^{Nm} \exp \left[ -\frac{mx}{(1 - \frac{1}{5m}) \Omega R} \right] dx
\]

\[
= \frac{2}{\Gamma(Nm)} \int_{0}^{\infty} Q(y)y^{(2Nm-1)-1} \exp \left[ -\frac{m y^2}{(1 - \frac{1}{5m}) \Omega R} \right] dy
\]

\[
= \frac{2}{\sqrt{\pi} (2Nm - 1)} \left[ \frac{m}{(1 - \frac{1}{5m}) \Omega R} \right]^{Nm}
\]

\[
\times 2F_1 \left( \frac{2Nm - 1}{2}; Nm; \frac{2Nm - 1}{2}; \left[ \frac{m}{(1 - \frac{1}{5m}) \Omega R} \right]^2 \right),
\]

Figure 4.7 compares the BER for different DS-PSK ultra wideband systems with \( N = 5, m = 2 \) and \( K = 5 \). Again, MRC shows the lowest BER among the three diversity techniques. It is evident that the processing gain (the number of chips in a symbol, \( N_s \)) has an important effect on the BER. The system performance increases when \( N_s \) is bigger. When \( N_s = 15 \), the data transmission rate could reach up to 95 Mbps, although the bit error rate is extremely high (about 10%).

In Figure 4.8, we evaluate the BER as a function of the diversity order \( N \) and the fading parameter \( m \) in a multiuser environment \( K = 5 \) at the data transmission rate of 96 Mbps. It is observed that DS-PSK ultra wideband systems are not sensitive to fading,
Figure 4.7: Average BER of DS-PSK as a function of $E_s/N_0$ and $N_s$. 
since no significant changes could be found when changing the parameter $m$. Obviously, increasing the diversity order is an appropriate way to improve the system performance in both MRC or EGC ultra wideband systems when using DS-PSK, comparing to selection diversity DS-PSK ultra wideband systems. In a severe interference environment, SD might not be the good candidate for DS-PSK ultra wideband signals.

Figure 4.8: Average BER of DS-PSK as a function of $N$ and $m$.

Figure 4.9 shows the average BER as a function of $K$ and $E_s/N_0$ at $m = 2$ and $N = 15$ by employing the MRC diversity technique. It is evident that increasing the transmitted signal power might not be feasible in reducing the effect of bit errors when more users are added to UWB systems.
Figure 4.9: Average BER of DS-PSK as a function of $E_s/N_0$ and $K$. 
4.4.3 Outage Probability for DS-PSK Ultra Wideband Systems

When using the SD technique, with the given threshold $\varepsilon$ the outage probability is given by

$$P_{\text{SD}}^{\text{out}} = \int_{0}^{\varepsilon} f_{\gamma_{\text{SD}}}(x) dx$$

$$= \sum_{n=0}^{N-1} \int_{0}^{\infty} \left( \frac{m}{\Omega_{1n} R_n} \right) \frac{x^{m-1}}{\Gamma(m)} \exp \left( - \frac{m x}{\Omega_{1n} R_n} \right) \prod_{i=0, i \neq n}^{N-1} \frac{G \left( m, \frac{m x}{\Omega_{1i} R_i} \right)}{\Gamma(m)} dx$$

$$= \frac{1}{\Gamma(m)} \sum_{n=0}^{N-1} \int_{0}^{\varepsilon_{m n}} x^{m-1} \exp (-x)$$

$$\times \prod_{i=0, i \neq n}^{N-1} \left[ 1 - \exp \left( - \frac{\Omega_{1i} R_i x}{\Omega_{1n} R_n} \right) \sum_{a=0}^{m-1} \left( \frac{\Omega_{1n} R_n x}{\Omega_{1i} R_i} \right)^a a! \right] dx. \quad (4.39)$$

Numerical values for (4.39) could be calculated by 16-point Gaussian-Laguerre integration, as shown in Appendix B.

When using the MRC technique, the outage probability is given by

$$P_{\text{MRC}}^{\text{out}} = \int_{0}^{\varepsilon} f_{\gamma_{\text{MRC}}}(x) dx$$

$$= \frac{1}{\Gamma(m_T)} \int_{0}^{m_T} \left( \frac{m_T}{R \Omega_T} \right)^{m_T} x^{m_T-1} \exp \left( - \frac{m_T x}{R \Omega_T} \right) dx$$

$$= \frac{1}{\Gamma(m_T)} \int_{0}^{\varepsilon_{m_T}} x^{m_T-1} \exp (-x) dx, \quad (4.40)$$

where the definitions of $m_T$ and $\Omega_T$ could be found in equation (2.39). By using [67, 3.381, 2], the above equations become

$$P_{\text{MRC}}^{\text{out}} = \frac{\exp \left( \frac{\varepsilon_{m_T}}{R \Omega_T} \right)}{\Gamma(m_T)} \sum_{a=0}^{\infty} (-1)^a \frac{\left( \frac{\varepsilon_{m_T}}{R \Omega_T} \right)^{a+m_T}}{m_T (m_T + 1) \cdots (m_T + a)}. \quad (4.41)$$
Similarly, when using the EGC technique, the outage probability becomes

\[ P_{\text{out}}^{\text{EGC}} = \int_{0}^{\varepsilon} f_{\gamma_{\text{EGC}}}(x) \, dx \]

\[ = \int_{0}^{\varepsilon} \frac{x^{N-1}}{\Gamma(Nm)} \left( \frac{m}{1 - \frac{1}{5m}} \Omega R \right)^{Nm} \exp \left[ -\frac{mx}{(1 - \frac{1}{5m}) \Omega R} \right] \, dx \]

\[ = \frac{1}{\Gamma(Nm)} \int_{0}^{\varepsilon} \left( \frac{1}{(1 - \frac{1}{5m}) \Omega R} \right)^{Nm} x^{N-1} \exp (-x) \, dx \]

\[ = \exp \left[ -\frac{\varepsilon m}{(1 - \frac{1}{5m}) \Omega R} \right] \sum_{a=0}^{\infty} (-1)^{a} \frac{\left( \frac{\varepsilon m}{(1 - \frac{1}{5m}) \Omega R} \right)^{a + Nm}}{Nm (Nm + 1) \cdots (Nm + a)}. \quad (4.42) \]

Figure 4.10 illustrates the outage probability with different system design parameters by using the EGC technique. The plot is obtained for \( m = 1, K = 5, N_s = 63 \) and

![Outage Probability Graph](image-url)
$E_s/N_0 = 10$ dB. Note that in order to achieve lower outage probability, the number of diversity branches should be carefully chosen to satisfy the desired SNIR. For example, at the threshold $\varepsilon = 0$ dB, to satisfy the outage probability less than $10^{-3}$, the diversity order $N$ should be greater than 3.

Notice that when increasing the number of active users, the outage probability increases, thus yielding the degradation of the system performance, as shown in Figure 4.11. Figure 4.11 also compares the effect of the processing gain $N_s$ on the outage probability using $m = 1$, $N = 10$ and $\varepsilon = 0$ dB. Increasing the processing gain $N_s$ results in decreasing the data transmission rate, but improving the multiple access capacity.

Figure 4.11: Outage probability of DS-PSK as a function of $E_s/N_0$, $K$ and $N_s$. 

103
4.5 Conclusions

In this chapter, the performance analysis of DS-PSK ultra wideband system is presented. DS is a well-known technique used in cellular communication systems and has been proposed for UWB systems. The key motivations for using the DS scheme are higher multiple access capacity and potential data transmission rate. In this chapter, the analysis methods used in cellular DS-CDMA systems were adopted for UWB communications and the performance analysis was performed in a multipath and multiuser environment. The followings can be concluded from this chapter:

(1) By using the DS multiple access scheme, a higher transmitting data rate could be achieved than in conventional TH UWB systems. For example, the bit rate could be about 100 Mbps in the given example. However, at a higher transmitting data rate, a DS-PSK ultra wideband suffers more self-interference comparing to conventional TH UWB systems.

(2) The processing gain has an important role in determining the system performance. The trade-off between the data transmitting rate \( R_b = \frac{1}{N_s T_p} \) and the system performance should be considered when choosing the value of the processing gain \( N_s \).

(3) In order to maintain high data transmission rate and improved system performance, diversity techniques should be used. Level of performance depends on the type of diversity. Obviously, using the MRC or EGC technique shows the performance improvement comparing to the SD scheme, which is consistent with expected results.

(4) DS-PSK ultra wideband systems are not very sensitive to the Nakagami fading parameter and the MIP. This implies that such systems could still provide reliable services in a severe fading environment.
(5) The use of rectangular pulse is not advisable in the DS-PSK ultra wideband systems.

Therefore, the numerical results are all based on Gaussian pulses.
5.1 Introduction

Based on the previous analysis, one may concluded that better performance can be achieved by using the TH UWB multiple access scheme, while the DS UWB multiple access scheme has potential to reach a higher data transmitting rate. To combine advantages of both schemes, the new technique, denoted as Direct Sequence-Time Hopping (DS-TH) is proposed in this dissertation.

The DS-TH is a hybrid spread spectrum techniques, which employs a hybrid signal format involving DS and TH. It is believed that the hybrid DS-TH can significantly reduce the acquisition time [72], thereby increasing the system performance. Particularly in UWB systems, due to the lower signal power, the receiver might need more processing time in order to accurately estimate the timing of transmitted signals. Therefore, the DS-TH multiple access scheme is a good candidate for UWB systems.

In this chapter, the transmitted and received UWB signal formats are introduced. This introduction is followed by the system performance evaluation for the three diversity combiners. To the best of our knowledge, this is the first report that investigates the system performance of the hybrid DS-TH ultra wideband systems.
5.2 DS-TH Ultra Wideband Signals

The hybrid DS-TH ultra wideband signal format is shown in Figure 5.1. Two levels of spreading are performed on the data symbols: first by the direct sequence spreading and then by the method of time hopping. A data symbol $d_{jk}$ is first spread by the DS symbol $b_{jk}$ with a processing gain $N_s$. Then each DS-chip is spread by the method of TH with $N_T$ hops per DS-chip with the hopping sequence $c_{jk}$.

![Figure 5.1: DS-TH ultra wideband signal format.](image-url)
By employing Pulse Shift Keying (PSK) technique, the typical signal modulation functions of the \( k \)th transmitting user are given by

\[
F(j, k) = b_{jk} (2d_{jk} - 1),
\]

\[
G(j, k) = jT_f + c_{jk}T_p,
\]

(5.1)

where \( b_{jk} \) is the DS spread PN code with \( b_{jk} \in \{-1, 1\} \), and the binary information \( d_{jk} \in \{0, 1\} \) is modulated by using PSK, resulting in \( 2d_{jk} - 1 \in \{-1, 1\} \). That means that the user bit 1 is transmitted by using pulses at zero degrees and a bit 0 is transmitted by using pulses at 180 degrees. In addition, \( c_{jk} \) is a distinct time hopping PN code set assigned to each user, which is periodic with each element chosen from the range \( \{0, 1, 2, \ldots, N_T - 1\} \); \( T_p \) is the TH chip duration, which is assumed to be the same as the pulse width; and \( T_f \) is the DS chip duration with \( T_f = N_T T_p \). The transmitted signal symbol from the \( k \)th user can be expressed as

\[
s_k(t) = \sum_{j=0}^{N_s} b_{jk} (2d_{jk} - 1) p_{tr}(t - jT_f - c_{jk}T_p - t_k),
\]

(5.2)

where the DS processing gain \( N_s \) is also the number of pulses in a symbol. Thus, the data transmission rate is \( R_b = \frac{1}{N_s T_f} \).

Signals from \( K \) users and \( L \) multipaths are summed at the receiver, resulting in

\[
r(t) = \sum_{k=1}^{K} \sum_{l=0}^{L-1} \sum_{j=0}^{N_s-1} A_{kl}b_{jk} (2d_{jk} - 1)
\]

\[
\times p_{rec}(t - jT_f - c_{jk}T_p - t_k - lT_p) + n(t).
\]

(5.3)

The reference signal, which is assumed to be perfectly synchronous with the desired signal, is given by
\[ s_1^*(t) = \sqrt{E_p} \sum_{i=0}^{N_s-1} b_{i1} p_{\text{rec}}(t - iT_f - c_{i1} T_p), \] (5.4)

with

\[ N_s E_p \theta_1 = 1 \] (5.5)

and \( \theta_1 = \int_{-\infty}^{\infty} p_{\text{rec}}^2(t) dt. \)

5.3 Variance of DS-TH Ultra Wideband Systems

At each diversity branch, the received signal is correlated with the reference signal. In this case, the general cross-correlation functions can be written as

\[ R_{n_n} (nT_p) = \sqrt{E_p} \int_0^{T_b} b_{j1} n(t) p_{\text{rec}}(t - iT_f - c_{i1} T_p - nT_p) dt, \] (5.6)

and

\[ R_{p_{\text{rec}} p_{\text{rec}}} (nT_p - lT_p) = \sqrt{E_p} \int_0^{T_b} b_{jk} (2d_{jk} - 1) \]
\[ \times b_{j1} p_{\text{rec}}(t - jT_f - c_{jk} T_p - t_k - lT_p) \]
\[ \times p_{\text{rec}}(t - iT_f - c_{i1} T_p - nT_p) dt \]
\[ = \sqrt{E_p} (2d_{jk} - 1) b_{jk} b_{j1} \int_0^{T_b} p_{\text{rec}}(t + \Delta) p_{\text{rec}}(t) dt, \] (5.7)

where \( \Delta = (i - j) N_T T_p + (c_{i1} - c_{jk}) T_p + (n - l) T_p - t_k. \)

At the \( n \)th branch, the desired term from user 1 over the \( l \)th multipath is given by

\[ z_{n1} = A_{1n} \sqrt{E_s E_p} \sum_{j=0}^{N_s} (2d_{j1} - 1) b_{j1} b_{j1} \int_0^{T_b} p_{\text{rec}}^2(t) dt \]
\[ = A_{1n} \sqrt{E_s E_p N_s} (\pm 1) \theta_1, \] (5.8)

with the factor \( b_{j1} b_{j1} = 1 \) and \( (2d_{j1} - 1) \in \{ \pm 1 \}. \)
Unlike DS-PSK ultra wideband systems, the DS-TH multiple access separates the pulses by using the TH method in order to reduce self-interference. However, the duration of $T_f$ might be less than that in TH UWB systems in order to maintain the required transmitting bit rate. Therefore, the effect of self-interference depends on the number of multipath components and spreading parameters. In this case, we have

$$z_{n2} = \sqrt{\frac{E_sE_p}{N_s}} \sum_{l=0,l \neq n}^{L-1} \sum_{j=0}^{N_s-1} \sum_{i=0}^{N_s-1} A_{1l} (2d_{j1} - 1) b_{j1} b_{i1} \times \int_0^{T_b} p_{rec} [t + (i - j)N_T T_p + (c_{i1} - c_{j1})T_p + (n - l)T_p] p_{rec}(t) dt. \quad (5.9)$$

Notice that the above value is nonzero only when $\Delta = 0$, such that

$$\Delta = \left(i - j - \left\lfloor \frac{l - n}{N_T} \right\rfloor \right) N_T T_p + \left[c_x - \text{mod} \left(\frac{l - n}{N_T}\right)\right] T_p = 0, \quad (5.10)$$

where $c_x = c_{i1} - c_{j1}$, $\lfloor x \rfloor$ rounds $x$ to the nearest integer toward zero, and $\text{mod} \ (x)$ is the reminder operator, yielding a value in the range of $[- (N_T - 1), \ N_T - 1]$.

The value of $z_{n2}$ could also be written as

$$z_{n2} = \sqrt{\frac{E_sE_p}{N_s}} \sum_{l=0,l \neq n}^{L-1} \sum_{j=0}^{N_s-1} \sum_{i=0}^{N_s-1} A_{1l} (2d_{j1} - 1) b_{j1} b_{i1} \left(\left\lfloor \frac{l - n}{N_T} \right\rfloor \right) \theta_1. \quad (5.11)$$

It should be mentioned that the above equation exists when and only when $c_x = \text{mod} \left(\frac{l - n}{N_T}\right)$ in (5.10). In this case, it can be shown that the probability of $c_x = \text{mod} \left(\frac{l - n}{N_T}\right)$ is given by

$$\Pr \left[c_x = \text{mod} \left(\frac{l - n}{N_T}\right)\right] = \frac{N_T - |x|}{N_T^2}. \quad (5.12)$$

Since $c_x$ is not the constant sequence, for the worst case, there are totally $L_x$ self-interferences between pulses inside a symbol, such that
\[ L_x = N_s \left[ \frac{L}{N_T} \right] + (N_T - 1). \]  

(5.13)

It is more meaningful to consider the average performance instead of the worst case scenario. By assuming uniform MIP and using the step as shown in (4.9)-(4.13), the conditional variance of \( z_{n2} \) becomes

\[
\text{var} \left[ z_{n2} \middle| c_x = \text{mod} \left( \frac{l - n}{N_T} \right) \right] = \sum_{l=0, l \neq n}^{L-1} E_s E_p \theta_1^2 \Omega_{1l}.
\]

(5.14)

After taking off the condition, the variance becomes

\[
\sigma_{z_{n2}}^2 = \frac{E_s E_p \theta_1^2}{N_T} \sum_{l=0, l \neq n}^{L-1} \Omega_{1l} \left\{ N_T - \text{mod} \left( \frac{|l - n|}{N_T} \right) \right\}.
\]

(5.15)

In a DS-TH ultra wideband system, the transmission time difference, \( t_k \) may span many frame time durations, such that

\[
t_k = \begin{cases} 
0 & k = 1, \\
J_k' N_T T_p + J_k T_p + \varepsilon_k & k \geq 2,
\end{cases}
\]

(5.16)

where \( J_k' \) is the integer rounding \( t_k \) to the nearest frame toward zero, \( J_k \) is an integer rounding \( (t_k - J_k' N_T T_p) \) to the nearest pulse toward zero, uniformly distributed over \([0, N_T - 1]\), and \( \varepsilon_k \) is the round error, uniformly distributed over \([0, T_p]\). Therefore, the value of \( \Delta \) in a DS-TH ultra wideband system becomes

\[
\Delta = (i - j) N_T T_p + (c_{i1} - c_{jk}) T_p + (n - l) T_p - J_k' N_T T_p - J_k T_p - \varepsilon_k
\]

\[
= (i - j - J_k' - x) N_T T_p + (c_x - y) T_p - \varepsilon_k,
\]

(5.17)

with

\[
x = \left\lceil \frac{l - n + J_k}{N_T} \right\rceil,
\]
\[ y = \text{mod} \left( \frac{l - n + J_k}{N_T} \right). \]  

(5.18)

The MAI effect exists only when \( i = j + J_k' + x \) and \( c_x = y \). Thus, at the \( n \)th branch, the value of \( z_{n3} \) due to the MAI effect can be written as

\[ z_{n3} = \sqrt{\frac{E_s E_p}{N_s}} \sum_{k=2}^{K} \sum_{l=0}^{L-1} A_{1l} W_k, \]  

(5.19)

with

\[
W_k = \left[ d_k^{-1} \sum_{i=0}^{x-1} b_{(i-x-J_k'+N_s)k} b_{i1} + d_k^0 \sum_{i=x}^{N_s-2} b_{(i-x-J_k')k} b_{i1} \right. \\
\left. + d_k^0 b_{(N_s-x-J_k'-1)k} b_{(N_s-1)k} \right] \theta_{31} \\
+ \left[ d_k^{-1} b_{(N_s-x-J_k'-1)k} b_{01} + d_k^{-1} \sum_{i=0}^{x-1} b_{(i-x-J_k'+N_s)k} b_{(i+1)1} \\
+ d_k^0 \sum_{i=J_k}^{N_s-2} b_{(i-x-J_k')k} b_{(i+1)1} \right] \theta_{32}, \]  

(5.20)

and

\[
\theta_{31} = \int_0^{T_p} p_{\text{rec}}(t - \varepsilon_k) p_{\text{rec}}(t) \, dt, \\
\theta_{32} = \int_0^{T_p} p_{\text{rec}}(t + T_p - \varepsilon_k) p_{\text{rec}}(t) \, dt. \]  

(5.21)

Similarly, in order to use the method presented in [70], (5.20) can be written as

\[
W_k = d_k^{-1} \sum_{i=0}^{x-1} b_{(i-x-J_k'+N_s)k} \left[ b_{i1} \theta_{31} + b_{(i+1)1} \theta_{32} \right] \\
+ d_k^0 \sum_{i=x}^{N_s-2} b_{(i-x-J_k')k} \left[ b_{i1} \theta_{31} + b_{(i+1)1} \theta_{32} \right] \\
+ d_k^0 b_{(N_s-x-J_k'-1)k} b_{(N_s-1)1} \theta_{31} \\
+ d_k^{-1} \sum_{i=J_k}^{N_s-2} b_{(i-x-J_k')k} b_{(i+1)1} \theta_{32}. \]  

(5.22)
This equation is similar to (4.19). Therefore, by using the same analysis presented in the previous chapter, as shown in (4.20)-(4.26). The conditional variance of $z_{n3}$ could be obtained as

$$\sigma^2_{z_{n3}|c_x, J_k} = \text{var} (z_{n3}|c_x, J_k) = E_sE_p (K - 1) \theta_3 \sum_{l=0}^{L-1} \Omega_{kl}, \quad (5.23)$$

with

$$\theta_3 = \frac{1}{T_p} \int_0^{T_p} (\theta_{31}^2 + \theta_{32}^2) \, d\varepsilon_k \quad (5.24)$$

After taking off the condition $c_x$,

$$\sigma^2_{z_{n3}|J_k} = \frac{E_sE_p (K - 1) \theta_3}{N_T^2} \sum_{l=0}^{L-1} \Omega_{kl} \left\{ N_T - \text{mod} \left( \frac{|l - n + J_k|}{N_T} \right) \right\}. \quad (5.25)$$

Since $J_k$ is uniformly distributed, the variance of $z_{n3}$ finally becomes

$$\text{var} (z_{n3}) = \frac{E_sE_p (K - 1) \theta_3}{N_T^2} \sum_{l=0}^{L-1} \Omega_{kl} \chi(J_k), \quad (5.26)$$

with

$$\chi(J_k) = \frac{\sum_{J_k=0}^{N_T-1} \left\{ N_T - \text{mod} \left( \frac{|l - n + J_k|}{N_T} \right) \right\}}{N_T}. \quad (5.27)$$

In summary, with the uniform MIP, the value of $R_n$ at each RAKE branch can now be written as

$$R_n = \left\{ \frac{\Omega}{N_sN_T} \sum_{l=0, l \neq n}^{L-1} \chi(0) + \frac{(K - 1) \theta_3 \Omega}{N_T^2N_s\theta_1^2} \sum_{l=0}^{L-1} \chi(J_k) + \frac{1}{2E_s/N_0\theta_1} \right\}^{-1}, \quad (5.28)$$

where $\chi(\cdot)$ is defined in (5.27).
5.4 System Performance for DS-TH Ultra Wideband Communications

5.4.1 Signal-to-Noise plus Interference Ratio

In a DS-TH ultra wideband system, due to the use of PSK, the average SNIR at the output of the diversity combiner is defined in (3.68). By substituting (2.35), (2.40) and (2.44) into (3.69), the average SNIR for SD, MRC and EGC are respectively given by

\[
\gamma_{SD} = \int_{-\infty}^{\infty} x f_{\gamma_{SD}}(x) \, dx \\
= \sum_{n=0}^{N-1} \int_{-\infty}^{\infty} \left( \frac{m}{\Omega R_n} \right)^m x^m \exp \left( -\frac{m x}{\Omega R_n} \right) \prod_{i=0, i\neq n}^{N-1} \frac{G \left( m, \frac{m x}{\Omega R_i} \right)}{\Gamma(m)} \, dx \\
= \sum_{n=0}^{N-1} \frac{\Omega R_n}{\Gamma(m+1)} \int_{-\infty}^{\infty} x^m \exp (-x) \times \prod_{i=0, i\neq n}^{N-1} \left[ 1 - \exp \left( -\frac{R_n}{R_i} x \right) \sum_{a=0}^{m-1} \frac{\left( \frac{R_n}{R_i} x \right)^a}{a!} \right] \, dx,
\]

\[
\gamma_{MRC} = \int_{-\infty}^{\infty} x f_{\gamma_{MRC}}(x) \, dx \\
= \int_{0}^{\infty} \frac{1}{\Gamma(m_T)} \left( \frac{m_T}{RN\Omega} \right)^{m_T} x^{m_T} \exp \left( -\frac{m_T x}{RN\Omega} \right) \, dx \\
= \frac{RN\Omega}{\Gamma(m_T+1)} \int_{0}^{\infty} y^{m_T} \exp (-y) \, dy \\
= RN\Omega,
\]

and

\[
\gamma_{EGC} = \int_{-\infty}^{\infty} x f_{\gamma_{EGC}}(x) \, dx \\
= \int_{-\infty}^{\infty} x^{Nm} \left( \frac{1 - \frac{m}{\Omega R}}{\Gamma(Nm)} \right)^{Nm} \exp \left( -\frac{m x}{(1 - \frac{1}{5m}) \Omega R} \right) \, dx \\
= \left( 1 - \frac{1}{\delta m} \right) RN\Omega.
\]
where $R$ is the average of $R_n$. Sample results are presented in Figure 5.2 through 5.5.

Figure 5.2 shows the effect of the spreading sequences on the average SNIR. These curves illustrate the average SNIR as a function of $E_s/N_0$, the DS processing gain $N_s$, and the number of hopping slots inside a frame $N_T$ by using MRC with $N = 5$ and $K = 10$. If there is a pulse is transmitted in a symbol ($N_s = 1$), the average SNIR is always less than 2 dB. At lower $E_s/N_0$ (less than 5 dB), different spreading parameters ($N_s = 7, 15, 31, 63$) show little difference in determining the average SNIR. However, at higher $E_s/N_0$, the average SNIR goes up more rapidly with greater $N_s$ or $N_T$. Notice that if DS-TH ultra wideband systems have the same value of $N_sN_T$, no much difference could be observed in
terms of the average SNIR. For example, a system with $N_s = 15$, $N_T = 20$ shows the same average SNIR with a system for $N_s = 31$, $N_T = 10$; and a system with $N_s = 15$, $N_T = 5$ shows the same average SNIR with a system with $N_s = 7$, $N_T = 10$. It is worthwhile pointing out that the data transmission rate is determined by $N_s N_T$. In Figure 5.2, $N_s = 1$, $N_T = 10$ yields the highest data transmission rate $R_b = 143$ Mbps.

At the transmitting bit rate of 11 Mbps, the SNIR performance in a DS-TH ultra wideband system shows similar characteristics with DS-PSK and TH-PSK ultra wideband systems. The trade-off between the average SNIR and the multiple access capacity is shown in Figure 5.3 with $m = 1$ and $N = 5$. Obviously, when the number of active users increases,
the average SNIR decreases, especially at higher $E_s/N_0$. In order to support 20 active users in this system, diversity combiners might be used to improve the SNIR performance. For example, when using EGC, the SNIR increases about 3 dB at $E_s/N_0 = 10$ dB. Of course, using MRC could be used to achieve higher average SNIR. Another approach to increase the multiple access capacity at a given transmitting bit rate is to increase the diversity order, as shown in Figure 5.4. It can be concluded that the average SNIR is improved by increasing the diversity order in any UWB system. When $K = 5$, the TH-PSK ultra wideband system shows the highest average SNIR. However, when $K = 20$, the DS-TH ultra wideband system shows the highest average SNIR. It is also observed that in a multiuser

Figure 5.4: Effect of the diversity order on the average SNIR.
environment \((K = 20)\), the DS-TH ultra wideband systems shows obvious higher average SNIR than the other two UWB systems. However, as the diversity order increase, this difference decreases.

Figure 5.5 compares the three multiple access schemes with the application to MRC at 1 Mbps, 11 Mbps and 33 Mbps respectively, using \(N = 5\) and \(K = 4\). At 1 Mbps data transmission rate, TH and DS-TH yield higher average SNIR and only little difference could be observed between these two multiple access schemes. At lower data transmission rates, three multiple access schemes have very close effects on the average SNIR. However, the degradation is so obvious in DS UWB systems, especially at higher data transmission rates.
rates. Furthermore, comparing TH-PSK to DS-TH ultra wideband systems at a given number of active users $K = 4$, one can see that TH-PSK ultra wideband systems always show higher average SNIR.

5.4.2 Bit Error Rate for DS-TH Ultra Wideband Systems

With the assumptions that the diversity branches are independent and the interferences plus noise are Gaussian distributed, in general, the average BER at the output of the diversity combiner is defined in (2.28) where the conditional BER is the Q-function. The average BER can be obtained by substituting $f_{\gamma}(x)$ from (2.35), (2.40) and (2.44) into (2.28).

If the selection diversity is applied, the average BER can be written as

$$P_b^{SD} = \int_0^\infty Q(\sqrt{\gamma}) f_{\gamma,SD}(x) dx$$

$$= \int_0^\infty Q(\sqrt{x}) \sum_{n=0}^{N-1} \left( \frac{m}{\Omega R_n} \right)^m x^{m-1} \frac{\Gamma(m)}{\Gamma(m)}$$

$$\times \exp \left( -\frac{mx}{\Omega R_n} \right) \prod_{i=0, i\neq n}^{N-1} G \left( \frac{m}{\Omega R_i} \right) \frac{\Gamma(m)}{\Gamma(m)} dx$$

$$= \sum_{n=0}^{N-1} \int_0^\infty \frac{\exp(-x) x^{m-1}}{\Gamma(m)} Q \left( \sqrt{\frac{\Omega R_n x}{m}} \right)$$

$$\times \prod_{i=0, i\neq n}^{N-1} \left[ 1 - \exp \left( -\frac{R_n x}{R_i} \right) \sum_{a=0}^{m-1} \left( \frac{R_n x}{R_i} \right)^a a! \right] dx. \quad (5.32)$$

If the maximal ratio combining is applied, the average BER becomes

$$P_b^{MRC} = \int_0^\infty Q(\sqrt{\gamma}) f_{\gamma,SD}(x) dx$$

$$= \int_0^\infty Q(\sqrt{x}) \frac{1}{\Gamma(Nm)} \left( \frac{m}{RN\Omega} \right)^N x^{N-1} \exp \left( -\frac{mx}{R\Omega} \right) dx$$

119
\[ P_{b}^{\text{MRC}} = \frac{2 (m/N^m)^{Nm}}{\Gamma(Nm)} \int_{0}^{\infty} [1 - \Phi(y)] y^{(2Nm-1)-1} \exp \left( -\frac{my^2}{R\Omega} \right) dy \]

\[ = \frac{2 (m/N^m)^{Nm}}{\sqrt{\pi} (2Nm - 1)} _2F_1 \left[ \frac{2Nm - 1}{2}; mT; \frac{2Nm - 1}{2}; \left( \frac{m}{\Omega R} \right)^2 \right]. \quad (5.33) \]

Similarly, if the equal gain combining is applied, the average BER is

\[ P_{b}^{\text{EGC}} = \int_{0}^{\infty} Q(\sqrt{\gamma}) f_{\gamma,\text{EGC}}(x) dx \]

\[ = \frac{1}{\Gamma(Nm)} \int_{0}^{\infty} Q(\sqrt{x}) x^{Nm-1} \left[ \frac{m}{(1 - \frac{1}{Nm}) \Omega R} \right]^Nm \]

\[ \times \exp \left[ -\frac{mx}{(1 - \frac{1}{Nm}) \Omega R} \right] dx \]

\[ = \frac{1}{\Gamma(Nm)} 2 \left[ \frac{m}{(1 - \frac{1}{Nm}) \Omega R} \right]^{Nm} \]

\[ \times \int_{0}^{\infty} Q(y) y^{(2Nm-1)-1} \exp \left[ -\frac{my^2}{(1 - \frac{1}{Nm}) \Omega R} \right] dy \]

\[ = \frac{2}{\sqrt{\pi} (2Nm - 1)} \left[ \frac{m}{(1 - \frac{1}{Nm}) \Omega R} \right]^{Nm} \]

\[ \times _2F_1 \left\{ \frac{2Nm - 1}{2}; Nm; \frac{2Nm - 1}{2}; \left[ \frac{m}{(1 - \frac{1}{Nm}) \Omega R} \right]^2 \right\}. \quad (5.34) \]

Figures 5.6 and 5.7 compare three UWB multiple access schemes for a system employing MRC with \( m = 2 \) and \( N = 5 \). Observe that the trade-off between the system performance and the data transmission rate is the major concern in UWB systems. The DS multiple access scheme has the highest potential to reach the high transmitting bit rate, but it shows the lowest system performance in terms of the average BER. On the contrary, using the TH multiple access technique improves the system performance, but it may not support the higher transmitting bit rate with the proposed design.
Figure 5.6: BER performance of the three multiple access schemes ($K = 5$).
Another important trade-off is between the system performance and the multiple access capacity, as shown in Figure 5.7. As expected, for a single user transmission, the average BER remains the same regardless of data transmission rates. Unexpectedly, BER comparison shows complicated results: when the number of active users is small, TH-PSK presents the lowest bit error rate and DS-TH takes the second place. However, as the number of active users increases, the BER goes up rapidly in TH-PSK ultra wideband systems, resulting in lower system performance than DS-TH ultra wideband systems. This indicates the performance of the proposed DS-TH multiple access is the best candidate in a multiuser environment.

Figure 5.7: Trade-off between the BER and the multiple access capacity ($E_s/N_0 = 10$ dB).
and multipath environment, while the TH-PSK technique shows better performance for the single-user transmission.

The average BER of DS-TH ultra wideband systems is presented in Figures 5.8 and 5.9 by using $N_s = 15$ and $N_T = 8$. Consistent with prior results, the BER performance is evaluated as the function of $E_s/N_0$, fading parameter $m$, type of diversity combiners, diversity order $N$, and number of active users $K$. All the curves in Figure 5.8 are drawn by using $N = 5$ and $K = 5$; all the curves in Figure 5.9 are drawn by using $E_s/N_0 = 5$ dB and $m = 1$ with the application of MRC. As expected, the bit error rate decreases (i.e., the
system performance increases) when decreasing $m$, using the MRC scheme, increasing the diversity $N$, and/or decreasing active users.

![Figure 5.9: Average BER of DS-TH as a function $K$ and $N$.](image)

5.4.3 Outage Probability for DS-TH Ultra Wideband Systems

In a DS-TH ultra wideband system, for a given threshold $\varepsilon$, the outage probability of SD is given by

$$P_{out}^{SD} = \int_0^\varepsilon f_{\gamma_{SD}}(x)dx.$$  \hspace{1cm} (5.35)

After substituting (2.35) into the above equation, the outage probability can be written as
The outage probability can be expressed as:

\[ P_{\text{out}}^{SD} = \sum_{n=0}^{N-1} \int_{0}^{\infty} \left( \frac{m}{\Omega R_n^d} \right)^n \frac{x^{m-1}}{\Gamma(m)} \exp \left( -\frac{m x}{\Omega R_n^d} \right) \prod_{i=0, i \neq n}^{N-1} G \left( m, \frac{m x}{\Omega R_n^d} \right) dx \]

\[ = \frac{1}{\Gamma(m)} \sum_{n=0}^{N-1} \int_{0}^{m/n} x^{m-1} \exp (-x) \times \prod_{i=0, i \neq n}^{N-1} \left[ 1 - \exp \left( -\frac{R_ax}{R_i} \right) \sum_{a=0}^{m-1} \frac{(R_ax/R_i)^a}{a!} \right] dx. \]  

(5.35)

When using MRC, the outage probability becomes

\[ P_{\text{out}}^{MRC} = \int_{0}^{\varepsilon} f_{\gamma_{\text{MRC}}}(x) dx \]

\[ = \frac{1}{\Gamma(Nm)} \int_{0}^{\varepsilon} \left( \frac{m}{R \Omega} \right)^{Nm} x^{Nm-1} \exp \left( -\frac{m x}{R \Omega} \right) dx \]

\[ = \frac{1}{\Gamma(Nm)} \int_{0}^{m/R \Omega} x^{Nm-1} \exp (-x) dx \]

\[ = \exp \left( -\frac{m}{R \Omega} \right) \sum_{a=0}^{\infty} (-1)^a \frac{(\frac{m}{R \Omega})^{a+Nm}}{Nm (Nm + 1) \cdots (Nm + a)}. \]  

(5.36)

If EGC is applied at the receiver, the outage probability can be written as

\[ P_{\text{out}}^{EGC} = \int_{0}^{\varepsilon} f_{\gamma_{\text{EGC}}}(x) dx \]

\[ = \frac{1}{\Gamma(Nm)} \int_{0}^{1 - \frac{m}{R \Omega}} x^{Nm-1} \exp (-x) dx \]

\[ = \exp \left[ -\frac{\varepsilon m}{(1 - \frac{m}{R \Omega})^R} \right] \sum_{a=0}^{\infty} (-1)^a \frac{\left( \frac{\varepsilon m}{(1 - \frac{m}{R \Omega})} \right)^{a+Nm}}{Nm (Nm + 1) \cdots (Nm + a)}. \]  

(5.37)

It should be pointed out that more numerical results could be obtained by using the above expressions. In Figure 5.10, the effect of multiple access capacity on the outage probability is evaluated for three multiple access schemes. The EGC technique and values of \( m = 1, E_s/N_0 = 10 \text{ dB} \) and \( \varepsilon = 5 \text{ dB} \) are used for this plot. It can be seen that the change of the outage probability of TH-PSK ultra wideband systems is widest among

125
three multiple access schemes. That indicates that TH-PSK ultra wideband systems are more sensitive to the number of active users $K$. In general, DS-TH ultra wideband systems have the better performance than DS-PSK and TH-PSK ultra wideband systems in most cases, but at a smaller number of active users. It is also noticed that the outage probability significantly drops down when increasing the diversity order from $N = 5$ to $N = 10$.

![Outage Probability Comparison](image)

**Figure 5.10:** Outage probability comparison of three multiple access schemes as a function of $K$ and $\varepsilon$.

5.5 Conclusions

In this chapter, a new multiple access scheme for UWB communication was proposed, denoted as DS-TH. The framework signal format and system performance analysis was presented. The key motivation for using the DS-TH scheme is to combine the advantages
of the DS and TH multiple access schemes. The following can be concluded from this chapter:

1. Like DS UWB systems, DS-TH ultra wideband systems were shown to have the potential to reach higher data transmission rates, while still maintaining the better BER performance. By separating pulses with the TH concept, the self-interference and multiple access interference are reduced.

2. DS-TH shows higher multiple access capacity than the TH-PSK systems, e.g., when the number of users increases, the DS-TH ultra wideband systems show better performance. The advantage of this design is to support the higher data bit rate, while the disadvantage would be more interference involved.

3. The design of spreading sequences has a significant effect on the system performance. In this dissertation, only the numerical values of processing gains are mentioned. It is believed that the properties of spreading sequences should be further studied.

4. Like other multiple access schemes, the diversity techniques play very important roles in determining the system performance. In general, using MRC is optimal when the receiver has the perfect knowledge of the channel characteristics, and increasing the diversity order significantly improve the system performance.

5. DS-TH ultra wideband systems are not very sensitive to fading and MIP. This implies that a DS-TH UWB system is more resistant to fading.
CHAPTER VI

SUMMARY AND CONCLUSIONS

6.1 Summary of Results

Previous studies of UWB multiple access schemes mainly focused on the time hopping scheme. In this dissertation, we investigated the DS scheme combined with TH scheme. In this study, the system performance was evaluated in a multipath and multiuser fading environment. The Nakagami fading model was considered for the analysis. The criteria for evaluating system performance were the average SNIR, BER and outage probability. Results obtained from this dissertation are consistent with published results and with characteristic behavior of UWB systems. Throughout the dissertation, the evaluation of the system performance shows consistent results with the following conclusions:

(1) The diversity order, $N$ has a significant effect on the average SNIR, BER and outage probability. Increasing the diversity order can improve the system performance. Due to the concern of the circuit complexity, it is not necessary to choose the maximal number of diversity branches to achieve the required system performance.

(2) Severity of fading has adverse effect on the system performance. However, the sensitivity to fading is different among the three types of multiple access schemes, as well as the three types of diversity techniques. Our results show that DS-TH ultra wideband systems are more resistant to the fading environment.
The capacity of multiple access techniques is a function of the number of active users, $K$. For three multiple access schemes, as the number of active users increases, the system performance decreases. The degree of decrease depends on the pulse modulation technique, the multiple access scheme and the transmitting bit rate.

As the transmitted signal power increases, the system performance increases too. However, at higher $E_s/N_0$, the improvement slows down.

The shape of pulses also plays an important role in determining the performance. Rectangular pulses are always used for the theoretical performance analysis due to their simplicity. However, a significant difference was found between rectangular pulses and Gaussian pulses. The results show that rectangular pulses could only be substituted for Gaussian pulses in TH UWB systems with lower data transmission rates.

The exponential MIP is considered as the most accurate method to model UWB signal propagation. However, these results show little difference between the uniform and exponential MIP at very small decay factor. This is not true at high decay factor. For simplicity, uniform MIP were used for numerical analysis.

### 6.2 Summary of Chapters

In Chapter I, the brief introduction of UWB was presented, including the advantages of ultra wideband, its applications in wireless communications, literature review and contribution of this dissertation.

In Chapter II, the preliminary parameters, definitions, system models, assumptions and basic concepts used in this dissertation were presented. The derivation of the PDF at the output of diversity combiner, including MRC, EGC and SD, was also presented.
In Chapter III, the previous study of the TH multiple access was extended with various pulse modulation techniques, namely PPM, PSK and PAM. Graphical results indicated that TH-PSK and TH-PAM ultra wideband systems had better system performance than TH-PPM ultra wideband systems.

Chapter IV examined the DS multiple access scheme in the context of UWB communications. The derivation was based on the methods of traditional DS-CDMA systems, but the properties of UWB signals were considered for the analysis. Unlike TH-PSK ultra wideband systems, DS-PSK showed potential to reach higher transmitting bit rates. Another advantage of the DS multiple access scheme would be its higher multiple access capacity.

In Chapter V, a new UWB multiple access scheme was proposed, denoted as DS-TH. The framework, system model and analytical expressions of the proposed system were presented. Results indicated that DS-TH combined the advantages of both TH and DS multiple access schemes. Comparing three multiple access schemes, it can be concluded that the TH technique provides the best service only when a few users are active in a UWB system. By using hybrid DS-TH, better performance, higher transmitting bit rate and higher multiple access capacity could be achieved.

6.3 Future Research Suggestions

UWB communication opens new areas of research due to its special characteristics compared to narrowband and wideband systems. A profusion of new research agendas can be identified based on this research, including:
The performance of systems in this dissertation was undertaken for simple binary UWB systems. It would be interesting to investigate $M$-ary pulse modulation schemes.

The general digital channel model was used for the system analysis. Many new measurement and simulation results for various fading environments have been published recently. Extension to a more realistic channel models could be considered in future research.

Besides interference from other users inside the same system, UWB signals will encounter interference from many sources, primarily from narrowband systems that occupy the same frequency bands. Therefore, interference from other existing narrowband systems in a particular propagation environment might be added to the system model when analyzing the performance of multiple access UWB systems.

There exists a wide variety of UWB receiver architectures. Optimizing the UWB receiver performance in a multiuser environment is still a challenge. The future receiver design should focus on lowering the cost, reducing the interference and improving the system performance.

Due to the use of extremely short pulses, time synchronization presents a major challenge. Receiver design incorporating advanced synchronization technique could be an excellent achievement. In this dissertation, perfect synchronization was assumed when analyzing the system performance. Thus synchronization and acquisition is left for further research.
REFERENCES


APPENDIX A

DERIVATION OF THE VARIANCE OF NOISE

In this appendix, the variance of noise in any UWB system is shown. The noise term \( z_{n4} \) in (2.20) is given by

\[
z_{n4} = \int_0^{T_b} n(t)s_1^*(t),
\]

(A.1)

where \( s_1^*(t) = \sum_{i=0}^{N_s-1} P_{i1}^* (t - nT_p) \) is the reference signal at the receiver, defined in (3.4), (3.59), (4.4) and (5.4) for each UWB system; and \( n(t) \) is the white Gaussian noise with two-sided power spectral density \( \frac{N_0}{2} \). The mean of \( z_{n4} \) is

\[
E [z_{n4}] = \sum_{i=0}^{N_s-1} \int_0^{T_b} E [n(t)] P_{i1}^* (t - nT_p) dt = 0,
\]

(A.2)

and the variance of \( z_{n4} \) is

\[
\sigma_{z_{n4}}^2 = \text{var} (z_{n4})
\]

\[
= \sum_{i=0}^{N_s-1} \int_0^{T_b} \int_0^{T_b} E [n(t)P_i^* (t - nT_p) n(\lambda) P_{i1}^* (\lambda - nT_p) dt d\lambda
\]

\[
= \sum_{i=0}^{N_s-1} \int_0^{T_b} \int_0^{T_b} E [n(t)P_i^* (t - nT_p) \frac{N_0}{2} \delta(t - \lambda) P_{i1}^* (t - nT_p) P_{i1}^* (\lambda - nT_p) dt d\lambda
\]

\[
= \frac{N_0}{2} \sum_{i=0}^{N_s-1} \int_0^{T_b} [P_{i1}^* (t - nT_p)]^2 dt.
\]

(A.3)
Since the energy of the reference signal is normalized to one, i.e., Thus,

\[
\int_0^{T_b} [s_1^*(t)]^2 dt = 1,
\]

\[
\sum_{i=0}^{N_s-1} \int_0^{T_b} [P_{ii}^* (t - nT_p)]^2 = 1,
\]

(A.4)

(A.3) can be written as

\[
\sigma_{\text{n4}}^2 = \frac{N_0}{2}.
\]

(A.5)
APPENDIX B

GAUSS-LAGUERRE INTEGRATION

In this appendix, it is shown that the calculation of the average BER by using the 16-point Gauss-Laguerre integration. Gauss-Laguerre integration is applied to replace a complicated integral function by a simple polynomial over sufficiently small intervals. Thus, \( \int_{0}^{\infty} \exp(-x)f(x)dx \) is equivalent to the sum of function evaluations at sample points multiplied by weight factors, given by [65]

\[
\int_{0}^{\infty} \exp(-x)f(x)dx = \sum_{n=1}^{N=16} w_n f(x_n),
\]

(B.1)

where \( f(x) \) is a complicated function, varied from diversity combining techniques and multiple access schemes, \( N = 16 \) is the number of sample points, \( x_n \) is the sample points and \( w_n \) is the weight factor. Table A.1 shows the values of \( x_n \) and \( w_n \) for 16-point Gaussian-Laguerre integration [66].

<table>
<thead>
<tr>
<th>( n )</th>
<th>Sample Point</th>
<th>Weight</th>
<th>( n )</th>
<th>Sample Point</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.08765</td>
<td>2.06152 \times 10^{-01}</td>
<td>9</td>
<td>9.43831</td>
<td>1.48446 \times 10^{-05}</td>
</tr>
<tr>
<td>2</td>
<td>0.46270</td>
<td>3.31058 \times 10^{-01}</td>
<td>10</td>
<td>15.44153</td>
<td>6.82832 \times 10^{-07}</td>
</tr>
<tr>
<td>3</td>
<td>1.14106</td>
<td>2.65796 \times 10^{-01}</td>
<td>11</td>
<td>19.18016</td>
<td>1.88102 \times 10^{-08}</td>
</tr>
<tr>
<td>4</td>
<td>2.12928</td>
<td>1.36297 \times 10^{-01}</td>
<td>12</td>
<td>23.51591</td>
<td>2.86235 \times 10^{-10}</td>
</tr>
<tr>
<td>5</td>
<td>3.43709</td>
<td>4.73289 \times 10^{-02}</td>
<td>13</td>
<td>28.57873</td>
<td>2.12708 \times 10^{-12}</td>
</tr>
<tr>
<td>6</td>
<td>5.07802</td>
<td>1.12999 \times 10^{-02}</td>
<td>14</td>
<td>34.58340</td>
<td>6.29797 \times 10^{-15}</td>
</tr>
<tr>
<td>7</td>
<td>7.07034</td>
<td>1.84907 \times 10^{-02}</td>
<td>15</td>
<td>41.94045</td>
<td>5.05047 \times 10^{-18}</td>
</tr>
<tr>
<td>8</td>
<td>9.43831</td>
<td>2.04272 \times 10^{-04}</td>
<td>16</td>
<td>51.70116</td>
<td>4.16146 \times 10^{-22}</td>
</tr>
</tbody>
</table>