HYPOTHESIS TESTING FOR THE PROCESS CAPABILITY RATIO

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Process Capability Indices are used in the industry as decision making tools. The value of the indices are estimated from a sample and should be qualified. Qualification of an estimate is provided in the form of a confidence interval for the estimate or from a hypothesis test. C_p and C_{pk} are two of the most widely used process capability indices in industry.

A hypothesis test for C_p has been developed by Kane (1986). The aim of this thesis is to develop a hypothesis test for C_{pk}. The hypothesis test for C_{pk} is developed from the approximate two sided confidence interval by Nagata and Nagahata (1992 & 1994). An Excel application has been developed which allows the user to compute cut-off and sample sizes for user-defined sampling plans. The application also computes C_p and C_{pk} and their confidence intervals from process data.

Approved: Dale T. Masel

Assistant Professor of Industrial and Manufacturing Systems Engineering

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Chapter 1: Introduction

1.1 Process capability Analysis

Every manufacturing process has variation associated with it. Since process variation can never be totally eliminated, the variability in a process should be minimized to improve product quality. Process capability analysis deals with the techniques used to understand the variability of a process and its effect on the product performance. Process capability analysis is an important engineering decision-making tool and has found application in a number of areas: as a criterion for vendor selection, reducing variability in a manufacturing process, specifying process requirements for new equipment, predicting how well the process will hold tolerances, assisting product designers in selecting or modifying a process and formulating quality improvement programs. Process capability analysis techniques have helped manufacturers control the quality of goods produced.

The result of a statistical process capability analysis is the process capability index. Process capability indices were first introduced by Juran [1]. Process capability indices are used to determine how well the output of a process meets the specification requirements set by the customer.

The C_p index and the C_{pk} index were the first indices developed. Although new indices like C_{pm}, C_{pmk}, $C_p(u,v)$ have been developed to provide additional information about the process, the majority of the organizations using process capability indices still use C_p and C_{pk}.

Process capability analysis has found widespread acceptance in industry. Xerox, AT&T Bell Laboratories and Motorola, Inc. are some of the corporations that are using process capability indices to monitor and improve the quality of their products. By 1991, all of the big three US automakers were using statistical control and process capability indices to monitor and improve product quality [1]. They also required their suppliers to provide proof of quality via process capability indices.

Manufacturers now require suppliers to provide process capability index values with supplied goods as part of the contract. This is also true of machine qualification and process capability studies. Therefore it is necessary to show that the process capability ratio meets or exceeds target values [2]. One can test this requirement using a hypothesis test, where the value of the index can be compared to a critical value on which a decision of rejection or acceptance can be made.

Usually when conducting the hypothesis test, the population mean and standard deviation are unknown and must be estimated from the data. The values of the indices are estimated by estimators like $\hat{C}_p$ and $\hat{C}_{pk}$. The ability of $\hat{C}_p$ and $\hat{C}_{pk}$ to reflect the true value of the index depends on the inherent variability of the process and the sampling plan. A sampling plan consists of a sample size and cut-off criteria by which a decision on the capability of the process can be made. High $\hat{C}_p$ and $\hat{C}_{pk}$ values are required to demonstrate process capability with small sample sizes. Often sample sizes are fixed without giving due consideration to the sampling variability of the data. Kane [3] developed a method by which the computed sample sizes account for the above-mentioned variability. Chou, et al. [4] provided tables for selecting sample sizes.

From a study of the literature on process capability indices, Kane [3] has developed a hypothesis test for C_p, as well as procedures for determining the sample size and cut-off criteria for C_p. Chou, et al. [4] provided tables for selecting sample sizes. But, little work has been done on the hypothesis test for C_{pk}. This may be because $\hat{C}_{pk}$ follows the joint distribution of two complex non-central t-distributions. Developing a hypothesis test for C_{pk} will aid in the decision-making process. With the hypothesis test it will also be possible to develop sampling plans for C_{pk}. This research will develop a hypothesis test for C_{pk} and a method to compute the sample size and critical value for C_{pk}.

1.2 Research Objective

The aim of this thesis was to develop a hypothesis test and sampling plan for the process capability ratio C_{pk} , which will enable suppliers to test process capability. This will enable a statistically based method of choosing the right sample sizes and critical values for demonstrating process capability. A computer program using Visual Basic for Application (VBA) was developed to calculate the sample size and cut-off values for any Type I and Type II error probabilities.

1.3 Research

The development of the hypothesis test for C_{pk} followed the method used by Kane to develop the hypothesis test for C_p. The approximate confidence intervals for C_{pk} developed by Nagata and Nagahata [5] (see equation 2.12) were used to arrive at the formula for a hypothesis test.

In the Operating Characteristic (OC) curve method developed by Kane, [3] the Type I and Type II probabilities must be fixed and coordinated with the sample size in order to establish a critical value that can be used to judge the capability of the process. In this thesis, the method was computerized so that the sample size and critical value for both C_p and C_{pk} could be generated for any given Type I and Type II error probabilities.

1.3.1 Develop the hypothesis test computations for C_{pk}.

This step involved the actual mathematical development of the hypothesis test computations for C_{pk}. The formulae are derived based on the two-sided confidence interval for C_{pk} developed by Nagata and Nagahata [4] (see equation 2.12). After the formulae had been developed inputs and outputs were identified for coding purposes.

1.3.2 Coding the OC Curve for the process capability ratio C_p.

The mathematical formulation for the computations of the OC curve has been developed by Kane [3]. This step involves coding the OC curve computations for C_p.

1.3.3 Coding the user interface.

The required inputs were identified and coded to provide the outputs, sample size and cut-off value.

1.3.4 Testing using Test data.

Testing and validation were conducted with data where the required results were known from the literature or were calculated. The output from the code was then validated against the known results.

1.4 Outline of the Thesis

Chapter 2 of the thesis deals with the literature review in the field of process capability indices C_p and C_{pk} . In Chapter 3 the development of the hypothesis test for C_{pk} is discussed. This chapter also discusses the program architecture. Chapter 4 includes the results and validation and Chapter 5 deals with the conclusions and recommendations based on the results of this research.

Chapter 2: Literature Review

2.1 Literature Review

In statistics, estimates are made about parameters of a population by taking samples from the population. The parameter estimate is a random variable and is called a statistic. Every parameter estimate has associated with it a particular distribution. The value of the estimate depends on several variables including sample size and sampling techniques.

This chapter is divided into four sections. The first section deals with process capability indices. The second section deals with statistical inference, which involves hypothesis testing and confidence intervals for process capability ratios. The third section deals with operating characteristic curves for process capability indices. The fourth section briefly outlines the entire capability analysis process.

2.2 Process Capability Indices

The science of process capability analysis, first introduced by Juran [1], began as a comparison of the process output distribution with the product tolerances. Frequency histograms, log plots and control charts were used to compare process data to product tolerances. Process capability indices were born out of the need for an index that could relate information from the various plots into a single value. Pearn, Kotz and Johnson [6] discussed the distributional properties of the three basic indices, C_p, C_{pk} and C_{pm}, and their estimators. A new index C_{pmk} was proposed, which was more sensitive to the departure of the process mean from the target value and thus able to distinguish between off-target and on-target processes.

The C_p index is the simplest capability index and is defined as follows [1]:

$$C_p = \frac{USL - LSL}{6\sigma} \qquad (2.1)$$

where

USL = Upper specification limit,

LSL = Lower specification limit, and

σ = process standard deviation.

An estimate of C_p is $\hat{C}_p$, which is given by:

$$\hat{C}_p = \frac{USL - LSL}{6\hat{\sigma}} \qquad (2.2)$$

where

$\hat{\sigma}$ = sample standard deviation.

Note that the only random variable in equation (2.2) is $\hat{\sigma}$. The sample standard deviation $\hat{\sigma}$, follows a chi-squared distribution. Therefore, Kane [3] concluded that the sampling distribution for $\hat{C}_p$ is also related to the chi-squared distribution.

From the definition of the process capability ratio C_p, it is apparent that C_p measures potential capability as defined by the actual process spread and does not consider the process mean [1], so it gives no indication of actual process performance.

C_p measures the spread of the specifications relative to the 6σ spread in the process. C_{pk} was created to compensate for this weakness.

The process capability ratio C_{pk} is defined by a three step procedure as follows [1]:

$$C_{pu} = \frac{USL - \mu}{3\sigma} \tag{2.3}$$

$$C_{pl} = \frac{\mu - LSL}{3\sigma} \tag{2.4}$$

$$C_{pk} = \min(C_{pl}, C_{pu}) = \frac{\min(\mu - LSL, USL - \mu)}{3\sigma} \tag{2.5}$$

where

μ = process mean.

An estimate of C_{pk} is $\hat{C}_{pk}$, which is given by:

$$\hat{C}_{pk} = \frac{\min(\hat{\mu} - LSL, USL - \hat{\mu})}{3\hat{\sigma}} \tag{2.6}$$

where

$\hat{\mu}$ = process sample mean.

Note that $\hat{C}_{pk}$ is a function of both $\hat{\sigma}$ and $\hat{\mu}$. Also, because of the min function, the distribution of $\hat{C}_{pk}$ involves the joint distribution of two non-central t-distributed random variables [1]. This distribution is very complex and makes determining the exact confidence interval on $\hat{C}_{pk}$ very difficult.

The formula for C_{pk} takes into account the process mean. Deviation from the process mean is reflected in the value of the index. The capability of one-sided specification limits can be determined by using the appropriate C_{pu} or C_{pl}. C_{pk} by itself does not adequately measure process capability. It is an inadequate measure of process centering. The three processes in Figure 2.1 [6] have their C_p and C_{pk} values tabulated in Table 2.1.

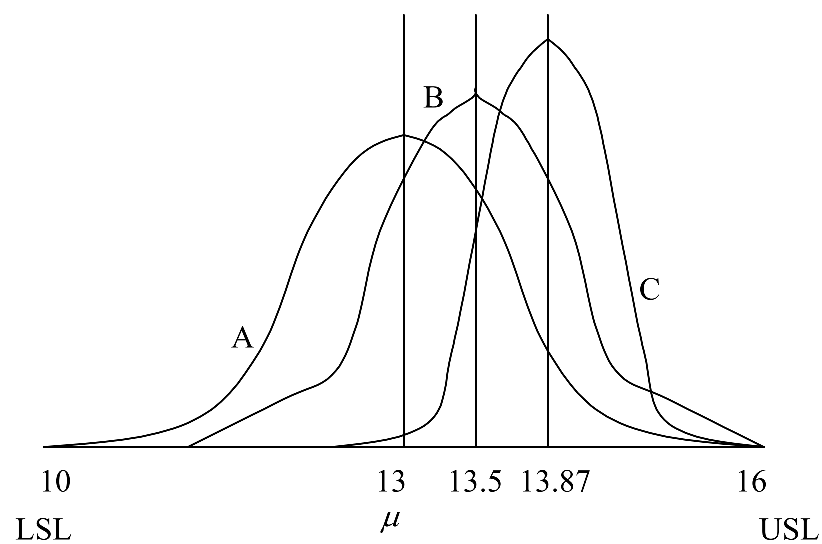


Figure 2.1: Three processes with C_{pk} =1 and their other Capability Indices.

Table 2.1: Three Processes with their C_p and C_{pk} values.

Process	μ	σ	C_p	C_{pk}
A	13.00	1.00	1.00	1.00
B	13.50	0.87	1.15	0.96
C	13.87	0.50	2.00	1.42

To characterize process centering satisfactorily, C_{pk} must be compared to C_p. C_p and C_{pk} together give a good indication of process capability with regard to both process spread and process location.

Processes are designed to achieve certain target values. Deviation from a target value can result in loss of quality and cost escalation [7]. C_p and C_{pk} do not account for departures from target values. Since their development, several new indices, such as C_{pm}, C_{pmk}, $C_p(u,v)$ have been developed which address the shortcomings of C_p and C_{pk}. The focus of this thesis, however, is on C_p and C_{pk} as they are the most widely used indices in industry [1].

2.3 Statistical Inference

In statistical inference, samples of data from a population are analyzed to draw conclusions about the population. Conclusions and inferences about population parameters are solely based on random samples. Since the samples result in statistics, which themselves are random variables, hypothesis testing is used to make statistical inferences on the sample statistic.

2.3.1 Statistical Hypothesis Testing

Statistical hypothesis testing involves a null hypothesis, H_0, and an alternate hypothesis, H_1. The rejection of H_0 leads to the acceptance of the alternate hypothesis, H_1. A null hypothesis concerning a population parameter is stated to specify an exact value of the parameter, whereas the alternative hypothesis allows for the possibility of a range of values. When a null hypothesis is accepted it means that there is not enough evidence to reject it.

Two errors are possible in hypothesis testing [8]:

Type I error (α): Rejecting H_0 when it is true.

Type II error (β): Accepting H_0 when it is false.

where

α and β denote the probability of occurrence of the Type I and Type II errors respectively [7].

A useful concept for evaluating the performance of a test is called the power of the test. The power of the test is its ability to correctly reject the null hypothesis. The power of the test, $Power(\theta)$, is defined as:

$$Power(\theta) = 1 - \beta(\theta) \tag{2.7}$$

where

θ = testing parameter or statistic

$\beta(\theta)$ = probability of a type II error for θ.

$\beta(\theta)$ should be minimized to increase the power of the test. Minimizing its value by increasing α to its largest acceptable value, or by increasing the sample size. In the ideal condition a departure from H_0 will be detected with certainty and the power of the test will be equal to 1. But for a fixed sample size, both α and β cannot simultaneously be minimized. Therefore, a small value of α is selected, and a rejection region, which minimizes β for the sample size n is calculated.

The purpose of using a process capability index is to judge whether a process is capable or not capable. This is a decision-making procedure. In process capability analysis the following hypothesis is tested.

H_0: The process is not capable,

versus the alternative hypothesis

H_1: The process is capable.

For the process capability ratio C_p, the hypotheses are as follows [3]

$$H_0 : C_p \leq c_0$$

$$H_1 : C_p > c_0 \tag{2.8}$$

where

c_0 is the standard minimal criterion for C_p.

The value c_0 is a lower bound process capability value and is usually decided by the organization depending upon the critical nature of the process. Kane [3] developed a hypothesis test for the process capability ratio C_p. Kane investigated this test and provides a table to assist calculation of sample sizes and critical values c, to test process capabilities.

Cheng [9] developed the hypothesis test for C_{pm} using the OC curve approach developed by Kane [3]. The impetus behind this development was to develop a decision-making procedure that was scientifically based. Chan, Cheng and Spiring [7] also used the OC curve approach to analyze $\hat{C}_{pm}$ when $\mu = T$ and a more generalized case where $(USL - T) \neq (T - LSL)$.

2.3.2 Confidence Intervals for Process Capability Ratios

Statistical estimation deals with parameter estimates of population parameters. A point estimate of a population parameter is a single value of the parameter, which is calculated from a random sample. It is unlikely for a point estimate to equal the population parameter. It is preferable to provide an interval estimate within which the value of the parameter will probably be found.

Considerable work has been done in the area of developing confidence intervals for process capability ratios. Several researchers have developed confidence intervals for the process capability estimators $\hat{C}_p$ and $\hat{C}_{pk}$.

Kane [3] provided the statistical base for developing the confidence interval for $\hat{C}_p$.

A $100(1-\alpha)\%$ confidence interval for C_p is given by [1]

$$P\left[\frac{\chi^2_{\alpha/2}}{\sqrt{n-1}}\hat{C}_p < C_p < \frac{\chi^2_{1-\alpha/2}}{\sqrt{n-1}}\hat{C}_p\right] = 1-\alpha \quad \text{with } (n-1) \text{ d.o.f.} \qquad (2.9)$$

The construction of confidence intervals for C_{pk} is difficult because the distribution of C_{pk} involves the joint distribution of two non-central t-distributed random variables. Several authors have constructed approximate confidence intervals by making various assumptions about the distribution of the point estimator $\hat{C}_{pk}$. Chou, Owen and Borrego [4] derived the lower confidence limit for C_{pk} by working with the exact distribution of $\hat{C}_{pk}$.

Kusher and Hurley [10] suggested a formula for the lower 100(1-α)% limit by using the normal approximation for the sampling distribution of $\hat{C}_{pk}$.

$$\hat{C}_{pk}\left[\frac{1-z_{1-\alpha}}{(2n-2)^{1/2}}\right] \tag{2.10}$$

The $100(1-\alpha)$% two-sided confidence interval suggested by Heavlin [11] is given by:

$$\left(\hat{C}_{pk} \pm z_{1-\alpha/2}\left\{\frac{n-1}{9n(n-3)}+\hat{C}_{pk}^{2}\,\frac{1}{2(n-3)}\left(1+\frac{6}{n-1}\right)\right\}^{1/2}\right) \tag{2.11}$$

Nagata and Nagahata [5], [12] developed an approximate two sided confidence interval for C_{pk} which is given by

$$\left(\hat{C}_{pk}-z_{\alpha/2}\sqrt{\frac{\hat{C}_{pk}^{2}}{2f}+\frac{1}{9n}},\hat{C}_{pk}+z_{\alpha/2}\sqrt{\frac{\hat{C}_{pk}^{2}}{2f}+\frac{1}{9n}}\right) \tag{2.12}$$

with $f=n-1$.

The approximate formula developed by Nagata and Nagahata (see equation 2.12 above) was used to develop the hypothesis test for C_{pk}.

2.4 Operating Characteristic Curve

The β error is the operating characteristic of the hypothesis test and is, in general, a function of the test parameter, θ [8]. The operating characteristic (OC) curve plots $\beta(\theta)$ versus the parameter θ. The OC curve is a graphical representation of the probability of committing a Type II error.

Kane [3] developed the OC curve for the case where the test parameter θ is the process capability ratio C_p. The sample variance follows a chi-squared distribution. Kane used this distribution to study the sampling variation in $\hat{C}_p$. The power equation developed by Kane is:

$$Power(C_p) = P[\hat{C}_p \rangle c] \tag{2.13}$$

$$Power(C_p) = P\left[\chi^2_{n-1} \langle \frac{C_p^2}{c^2}\right] \tag{2.14}$$

The β error is calculated using the equation

$$\beta(C_p) = 1 - Power(C_p) \tag{2.15}$$

The power is a critical component in computing the sample size and critical value in a sampling plan. It is the tradeoff between type I and type II errors that provide the two pieces of information necessary to determine the sample size and cut-off criteria.

Using equation (2.15) the β error is computed at different C_p values. Plotting the different β error values versus the corresponding C_p values gives the operating characteristic curve. By selecting α and β error values and the acceptable and rejectable quality levels for C_p, Kane [3] determined the required sample size and critical value for a hypothesis test. In other words, different sample sizes and critical values result in different OC curves. For example, by stating a minimum acceptable capability level, $C_p(low)$, that will be accepted with probability α and a maximum rejectable capability level, $C_p(high)$ that will be rejected with probability β, one can restate equations (2.9) and (2.14) to obtain:

$$\frac{C_p(high)}{C_p(low)} = \sqrt{\frac{\chi^2_{(n-1)}(1-\beta)}{\chi^2_{(n-1)}(\alpha)}} \tag{2.16}$$

$$\frac{c}{C_p(low)} = \sqrt{\frac{n-1}{\chi^2_{(n-1)}(\alpha)}} \tag{2.17}$$

It is now possible to determine the sample size and critical value from equations (2.16) and (2.17). The operating characteristic curve (OC) for C_p is shown below in Figure 2.2.

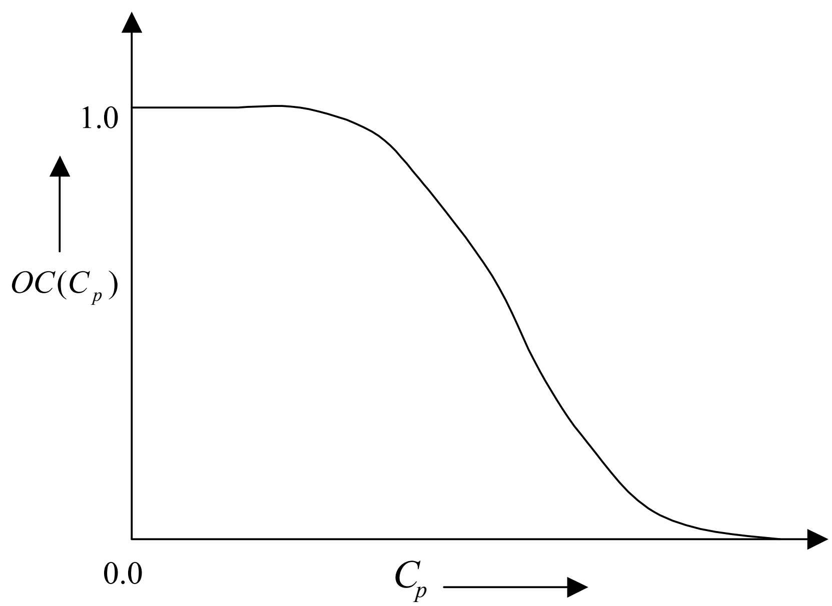


Figure 2.2: Operating characteristic curve for C_p.

In industry, sample sizes between 30 and 50 are commonly used for machine qualification studies [2]. The sample sizes are chosen arbitrarily. The OC curve method can be used to compare different testing schemes.

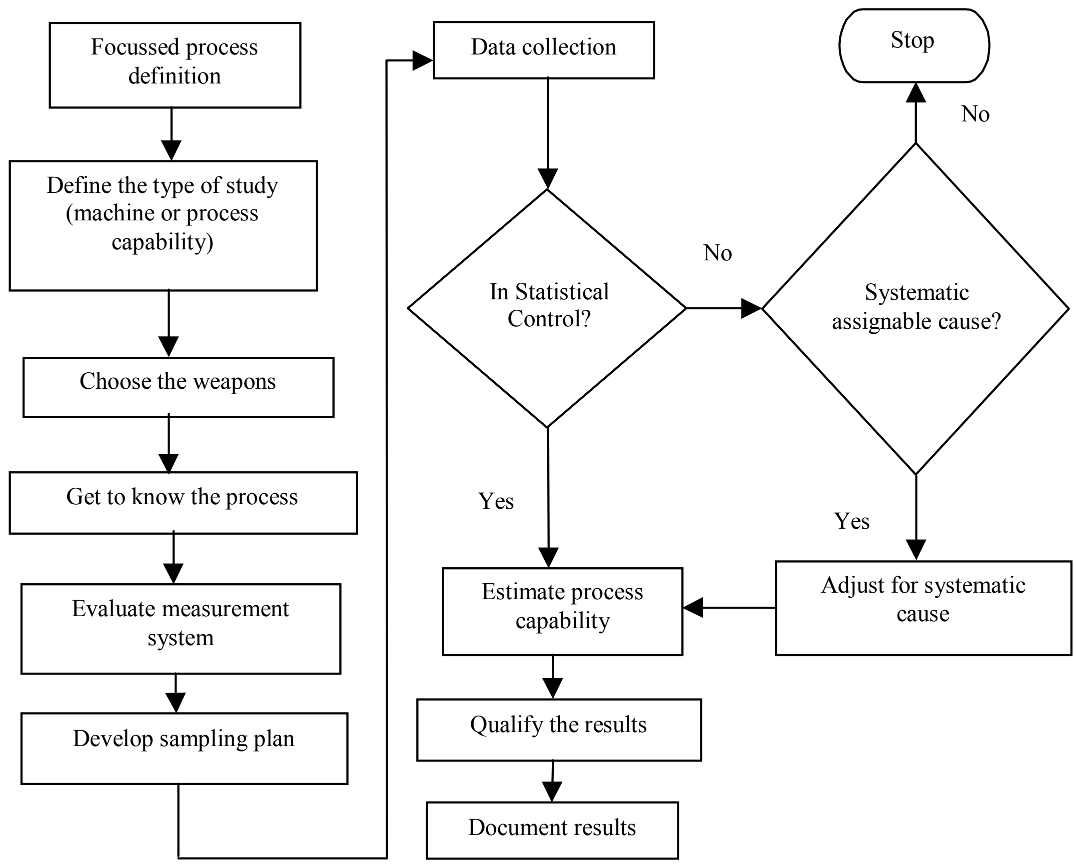


Figure 2.3: The process capability Analysis Process.

2.5 The Process Capability Analysis process.

The process capability analysis process involves the following steps as developed by Kotz and Lovelace [1].

Focussed process definition:

A product is usually manufactured in more than one step and often is also defined by more than one critical characteristic. The analyst therefore has to choose from more than one manufacturing process. A single process and parameter, which will indicate the overall outgoing quality of the product, is usually chosen after a careful study of the product requirements.

Define type of Capability Study – Machine or Process:

Machine capability studies concern the machines along with the fixtures, tools and gauges that are used in the manufacturing process. Machine capability studies are essential for qualifying new equipment, studying the effects of modifications on a machine and comparing machine capability. A machine capability study reflects a machine's repeatability on a short-term basis. On the other hand, the purpose of a process capability study is to determine the feasibility of the entire process to produce quality products. It involves normal operating conditions, operators who typically run the equipment and regular production raw material. Here the process reproducibility is measured over a longer period of time. Because additional sources of variability are present, process capability is usually lower than machine capability.

Choose the Indices:

This process involves selecting the indices by which to estimate capability. There are many indices to choose from but C_p, C_{pk} and C_{pm} are the most popular. Often C_p and C_{pk} are used together. Using the two indices together gives a better perspective on capability versus performance versus centering. A choice has to be made between univariate and multivariate indices depending upon the nature of the product and the process. The choice of index is usually governed by the customer's demand.

Practitioner-process bonding: get to know the process:

This is a very important step in the capability analysis process. Understanding the process helps to identify which dimension of the process output truly reflects the process behavior.

Evaluate the measuring system:

The data collected for analysis should be free of measurement error. Measurement error affects the performance of process capability indices. Measurement error is of two types. Bias results when the measured variable is constantly offset by a fixed amount. Measurement errors occur when the measurement system adds variability to the measurement process and incorrectly measures the variable. When bias is encountered the average of the process is affected. This significantly affects C_{pk} and C_{pm} but C_p is not affected. The presence of stochastic measurement error always results in a decrease in the estimates of C_p, C_{pk} and C_{pm}.

Develop the sampling plan:

It is critical to develop a sampling plan because the accuracy of the process capability estimates is dependent on the sampling size and the method used for sampling. Inadequate sample sizes result in poor estimates of the indices, which results in incorrect interpretations about the process. The process capability standard developed by the ANSI Z1 Committee on Quality Assurance (1996) outlines suggestions for developing sampling plans. The ability of $\hat{C}_p$ and $\hat{C}_{pk}$ to reflect the true value of the index depends on the sample size and the sampling plan. High $\hat{C}_p$ and $\hat{C}_{pk}$ values are required to demonstrate process capability with small sample sizes. Larger samples result in more precision in estimation and consequently a shorter confidence interval.

Data collection:

The procedure for the data collection process has to be documented correctly. Non-normal data, cyclical patterns and correlation in the data should be noted as well as the conditions under which the process was run. Operating conditions include the operator shift, etc.

Verify statistical control:

Control charts are used to verify the stability of a process. The $\overline{X}$ and R charts are commonly used in capability studies as they include both time-to-time variability and random error of the process. Process capability and the effectiveness of the control charts are related to each other, which further provides a relation between process capability and control chart sensitivity. In cases where subgroups larger than one are not possible, Moving Range, Cusum or EWMA charts are used to determine statistical control. Process capability should not be estimated when the process is not in control.

Estimate the process capability:

The process capability index can be calculated once the data has been collected and characterized and checked for statistical control.

Qualify the results:

A point estimate of a process capability index must be qualified by either confidence intervals or a hypothesis test. This ensures that the results are not misinterpreted. Also, the managers using the results should be aware of the sampling conditions under which the estimates were calculated.

Document results:

The estimate calculations as well as the analysis should be documented. Documentation of the test conditions, deviations from standard assumptions and sampling should be included in the report. The documentation should also include the confidence intervals and hypothesis test results along with the process capability index. Documentation is required for future studies and investigations.

Chapter 3: A Hypothesis Test for C_{pk}

3.1 Hypothesis Testing

Hypothesis testing and confidence interval techniques of statistical inference are closely related to each other. Confidence interval estimation involves calculating confidence limits within which the parameter in question lies with a certain probability. For a population with mean μ and known σ^2, the hypothesis test and confidence interval estimation is based on the random variable $Z = \dfrac{\overline{X} - \mu}{\sigma / \sqrt{n}}$.

The testing of the hypothesis $H_0 : \mu = \mu_0$ versus the hypothesis $H_1 : \mu \neq \mu_0$ at a significance level α is equivalent to computing a $100(1-\alpha)\%$ confidence interval on μ and rejecting H_0 if μ_0 is not inside the confidence interval. If μ_0 is inside the confidence interval then the hypothesis is not rejected.

3.2 Development of a Hypothesis test for C_{pk}

Developing a confidence interval for the process capability ratio C_{pk} was difficult, because the sampling distribution of C_{pk} follows a non-central t distribution. Several approximate confidence intervals have been developed for C_{pk} and were mentioned in the previous chapter. For the development of the hypothesis test for C_{pk} the confidence interval formulated by Nagata and Nagahata [5] was used. Nagata and Nagahata developed the confidence interval based on the normal distribution, given by:

$$\hat{C}_{pk} - z_{\alpha/2}\sqrt{\frac{\hat{C}_{pk}^{\;2}}{2f} + \frac{1}{9n}} \langle C_{pk} \langle \hat{C}_{pk} + z_{\alpha/2}\sqrt{\frac{\hat{C}_{pk}^{\;2}}{2f} + \frac{1}{9n}} \tag{3.1}$$

The hypothesis for C_{pk} is developed similarly to the hypothesis developed for C_p by Kane [3].

$$H_0 : C_{pk} \le c_{ko} \tag{3.2}$$

$$H_1 : C_{pk} > c_{ko} \tag{3.3}$$

The sample size and the cut-off were calculated by selecting α and β error values and the acceptable and rejectable quality levels for C_{pk}. Figure 3.1 and Figure 3.2 illustrate the calculations for sample size and cut-off values.

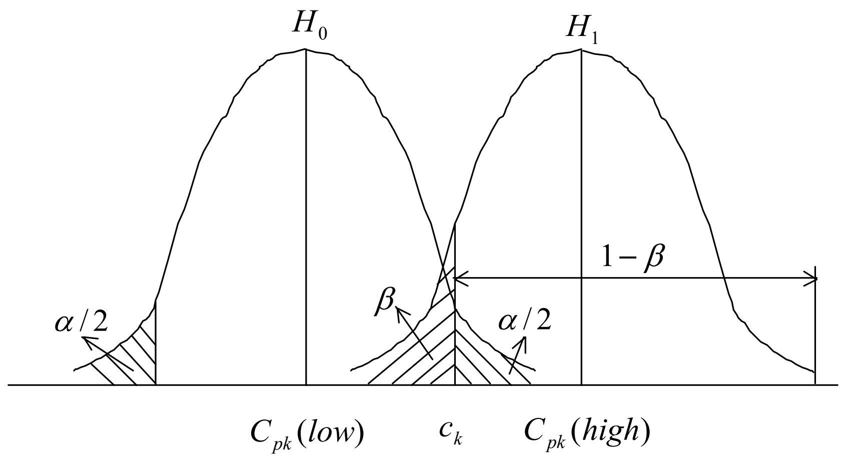


Figure 3.1: Calculation of cut-off and sample size value for C_{pk}.

For calculating c_k we use the Upper Specification Limit for C_{pk} (low)

$$c_k = \hat{C}_{pk}(low) + z_{\alpha/2}\sqrt{\frac{\hat{C}_{pk}\ (low)^2}{2f} + \frac{1}{9n}} \tag{3.4}$$

From Figure 3.1

$$c_k = \hat{C}_{pk}\ (high) - z_\beta \sqrt{\frac{\hat{C}_{pk}\ (high)^2}{2f} + \frac{1}{9n}} \tag{3.5}$$

$$z_\beta = \frac{c_k - \hat{C}_{pk}(high)}{\sqrt{\frac{\hat{C}_{pk}(high)^2}{2f} + \frac{1}{9n}}} \tag{3.6}$$

Substituting equation (3.4) in equation (3.6) results in the following equation:

$$z_\beta = \frac{\hat{C}_{pk}(low) + z_{\alpha/2}\sqrt{\frac{\hat{C}_{pk}\ (low)^2}{2f} + \frac{1}{9n}}}{\sqrt{\frac{\hat{C}_{pk}(high)^2}{2f} + \frac{1}{9n}}} \tag{3.7}$$

In equation (3.7) n is the only unknown variable. For a given α , β , AQL and RQL the value of the sample size n can be calculated. Once the value of n is known, substitution in equation (3.4) yields the cut-off value.

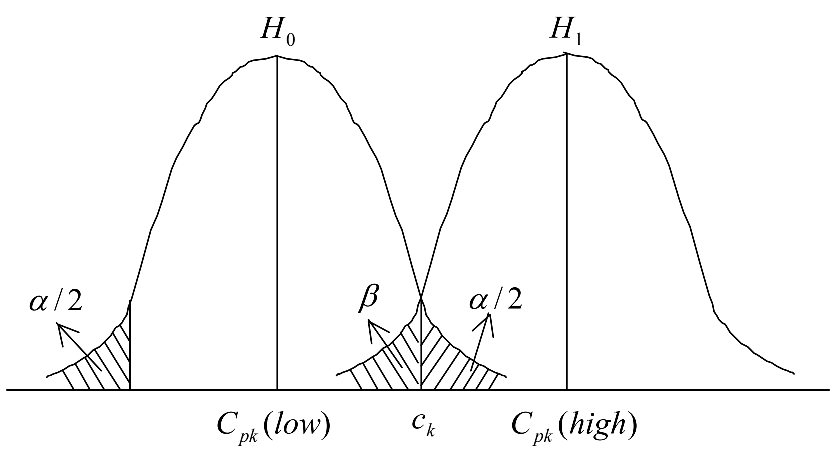


Figure 3.2: Calculation of cut-off and sample size value for C_{pk} .

The cut-off value c_k can only be a single value. The spread of the distribution is a function of the sample size n. By placing c_k with α and β probabilities on either side we can calculate the cut-off value.

From Figure 3.2

$$c_k = \hat{C}_{pk}\ (low) + z_{\alpha/2}\sqrt{\frac{\hat{C}_{pk}(low)^2}{2f} + \frac{1}{9n}} \tag{3.8}$$

If the cut-offs for the intervals coincide then

$$c_k = \hat{C}_{pk}(high) - z_{\beta}\sqrt{\frac{\hat{C}_{pk}(high)^2}{2f} + \frac{1}{9n}} \tag{3.9}$$

Therefore from equation (3.8) and (3.9)

$$\hat{C}_{pk}(low) + z_{\alpha/2}\sqrt{\frac{\hat{C}_{pk}(low)^2}{2f} + \frac{1}{9n}} = \hat{C}_{pk}(high) - z_{\beta}\sqrt{\frac{\hat{C}_{pk}(high)^2}{2f} + \frac{1}{9n}} \tag{3.10}$$

This results in

$$\hat{C}_{pk}(low) + z_{\alpha/2}\sqrt{\frac{\hat{C}_{pk}(low)^2}{2f} + \frac{1}{9n}} - \hat{C}_{pk}(high) - z_{\beta}\sqrt{\frac{\hat{C}_{pk}(high)^2}{2f} + \frac{1}{9n}} = 0 \tag{3.11}$$

Again n is the only unknown quantity and can be calculated from equation (3.11). Once n has been calculated the cut-off value is determined from either equation (3.8) or equation (3.9).

3.2 Need for a program

As mentioned earlier Kane [3] has developed a hypothesis test for the process capability ratio C_p . Kane investigated this test and developed a table (Table 3.1) for calculating sample sizes and critical values [3].

Table 3.1: Sample size and cut-off Value Determination for Testing C_p [3].

Sample Size	$\alpha = \beta = 0.10$		$\alpha = \beta = 0.05$	
	$\frac{C_p(high)}{C_p(low)}$	$\frac{c}{C_p(low)}$	$\frac{C_p(high)}{C_p(low)}$	$\frac{c}{C_p(low)}$
10	1.88	1.27	2.26	1.37
20	1.53	1.20	1.73	1.26
30	1.41	1.16	1.55	1.20
40	1.34	1.14	1.46	1.18
50	1.30	1.13	1.40	1.16
60	1.27	1.11	1.36	1.15
70	1.25	1.10	1.33	1.14
80	1.23	1.10	1.30	1.13
90	1.21	1.10	1.28	1.12
100	1.20	1.09	1.26	1.11

The program eliminates the need for such a table. It calculates the sample size and critical values for C_p and C_{pk} for user defined values and also plots the OC curve for C_p . The program also calculates the process capability indices C_p and C_{pk} and confidence intervals from user entered data.

3.3 The User Interface

The interface is an Excel spreadsheet. The user interface is illustrated in the following Figures (Refer Figures 3.3 through 3.11). The Excel object model has been programmed using Visual Basic for Applications (VBA) to provide added functionality. The required data is entered in the designated cells and the results of the calculations are displayed on the sheet. The user interface consists of 2 Excel spreadsheets. The first sheet titled 'Hypothesis Test' computes the sample size and cut-off from the user defined sampling plans. The second sheet titled 'PCRs and Confidence Intervals' allows the user to input data from a process. $\bar{X}$ and $\bar{R}$ charts are then plotted to determine whether the process is in control or not. Once it is determined that the process is in control, the indices C_p and C_{pk} can be computed along with their confidence intervals.

3.3.1 The Use Interface: Hypothesis Test sheet

The Hypothesis Test Sheet is as shown in Figure 3.3 below.

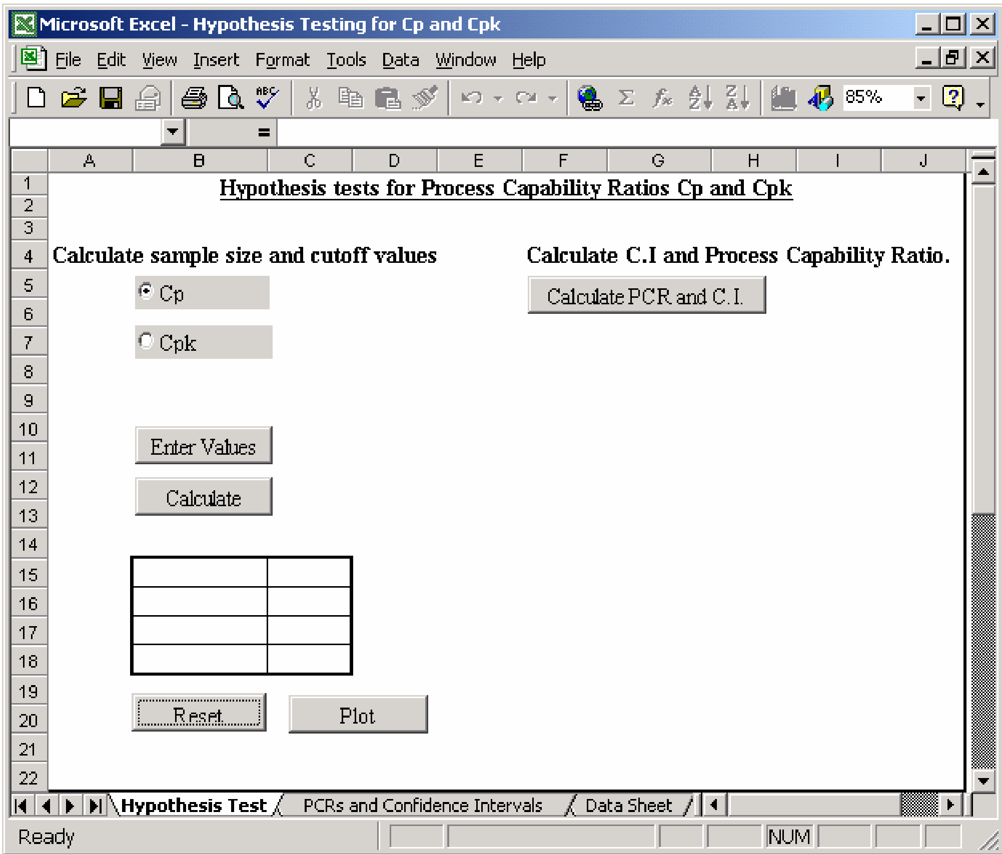


Figure 3.3: Hypothesis Test Sheet.

By clicking on the option button ‘Cp’ or option button ‘Cpk’ the user is able to select the process capability ratio for which the sample size and cut-off value has to be computed. Computing the cut-off value for a capability test requires specification of the acceptable quality level ($C_p(high)$ or $C_{pk}(high)$), rejectable quality level

($C_p(low)$ or $C_{pk}(low)$), and the alpha and beta error probabilities. The command button 'ENTER VALUES' prompts the user to input the required data as shown in Figure 3.4 below.

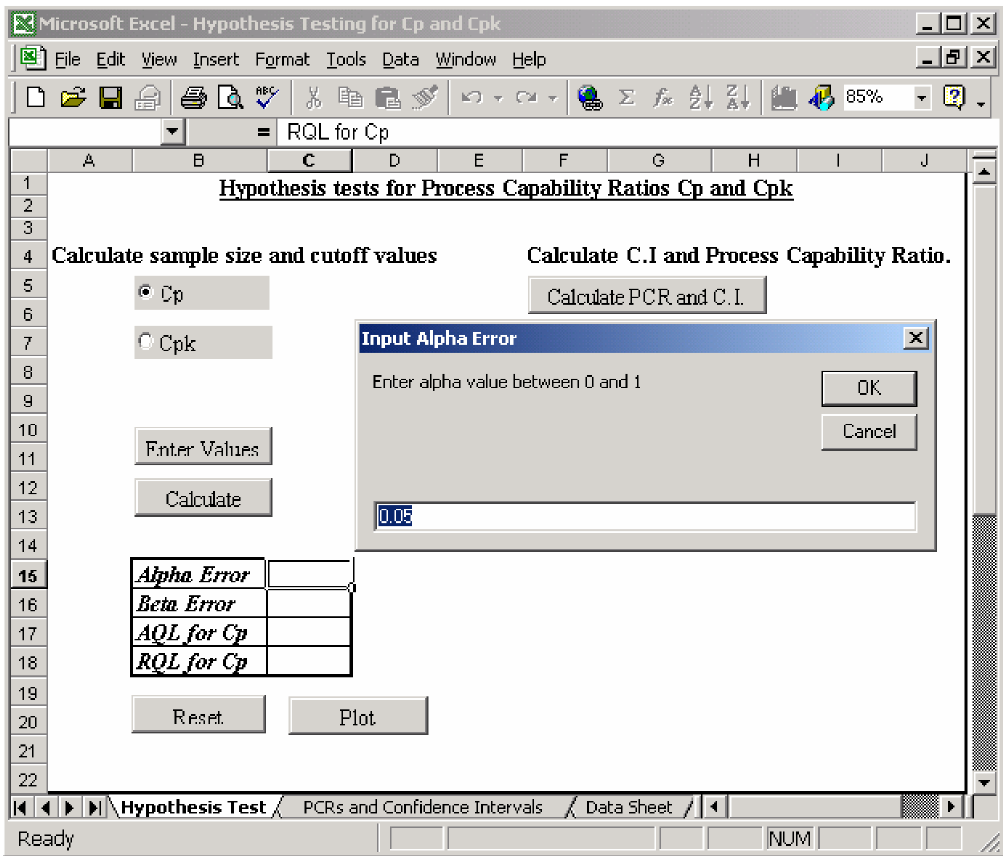


Figure 3.4: Data input for sampling plan.

There are a number of calculations that need to be performed before the solution is obtained. The command button 'CALCULATE' checks for valid data and then proceeds with the calculations. It runs the Excel object 'Goalseek' within Excel to arrive at the sample size. The cut-off value is then calculated from this sample size.

The calculations are performed in the background hidden from the user and the results are displayed on the sheet as shown in Figure 3.5 below.

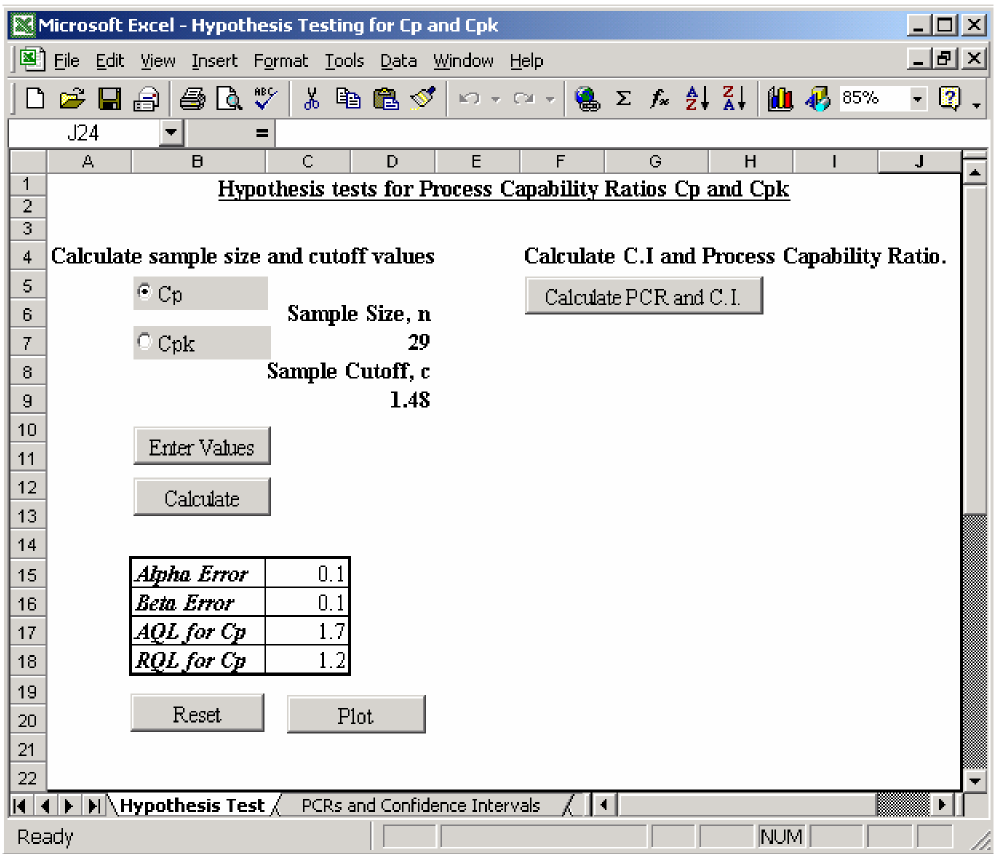


Figure 3.5: Display of sample size and cut-off value.

The command button 'PLOT' uses the sample size and cut-off values to generate a table to plot the OC curve for C_p for the user defined sampling plan. The table is hidden from the viewer. After the table has been generated the OC curve is plot. The OC curve for the particular sampling plan is displayed on a separate sheet as shown in Figure 3.6. The command button 'RESET' set all cells to initial state. The command button 'CALCULATE PCRs AND C.I.' brings up the PCRs and Confidence Interval sheet.

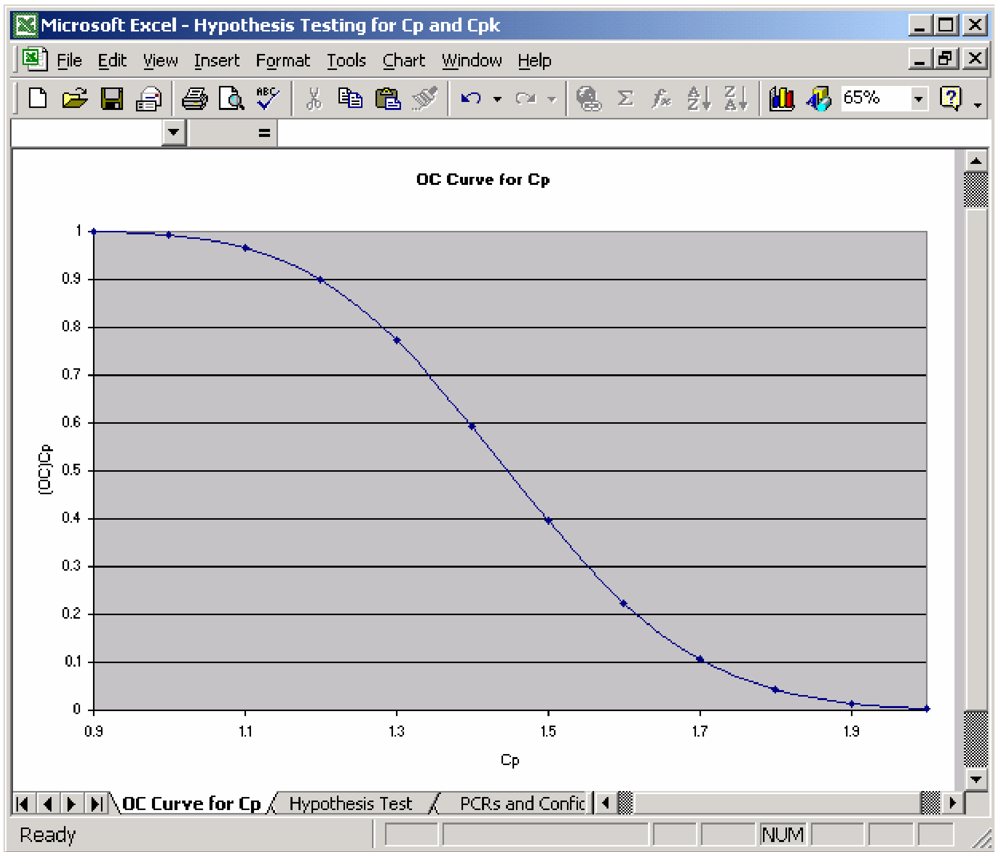


Figure 3.6: OC curve for C_p.

3.3.2 The User Interface: PCRs and Confidence Intervals

The PCRs and Confidence Intervals sheet is as shown below in Figure 3.7. The command button 'DATA TABLE' prompts the user for sample size and sub group size. A data table is created for the user defined sample size and sub group size. After the table is populated with data from an actual process, the sample mean and sample range values are computed by the command button 'CAL MEANS' as shown in Figure 3.8.

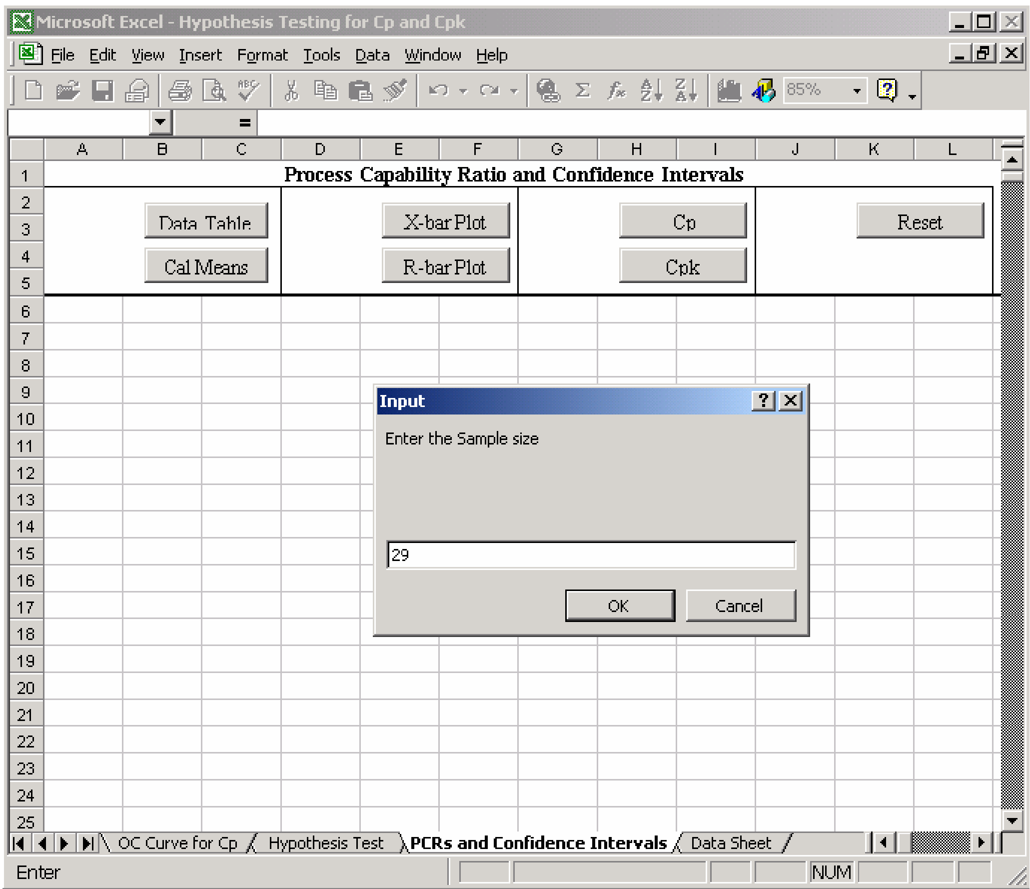
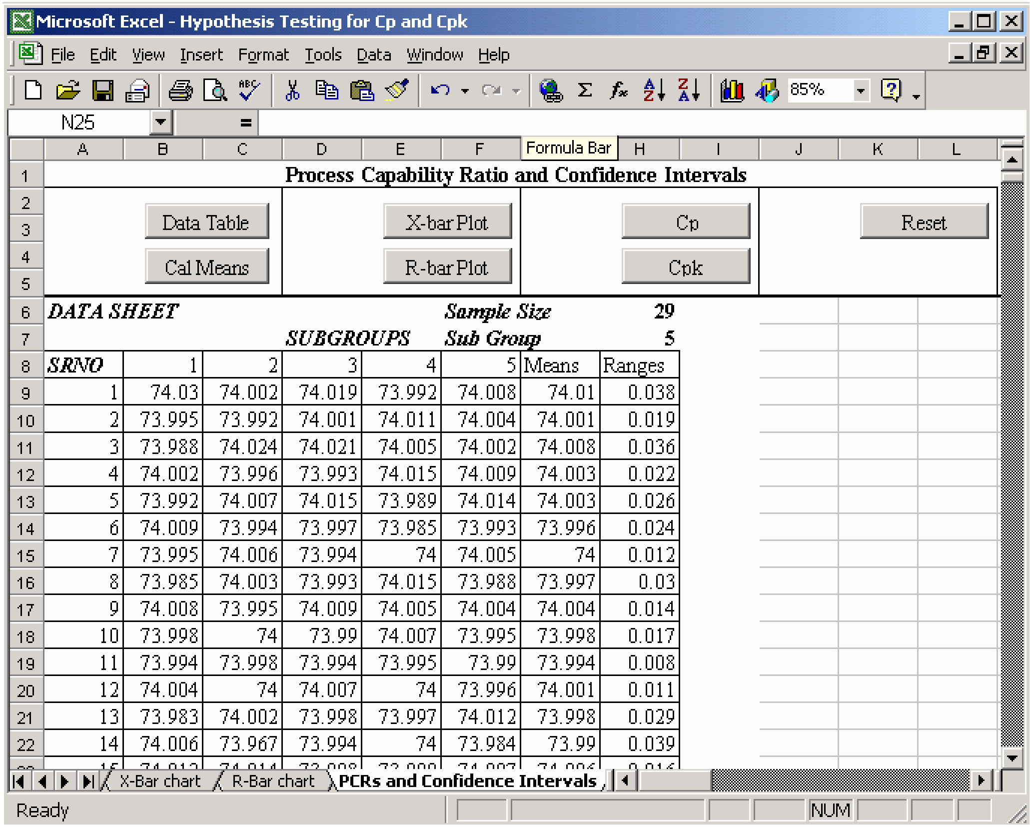


Figure 3.7: PCRs and Confidence Interval Sheet.



DATA SHEET				Sample Size		29	
			SUBGROUPS	Sub Group		5	
SRNO	1	2	3	4	5	Means	Ranges
1	74.03	74.002	74.019	73.992	74.008	74.01	0.038
2	73.995	73.992	74.001	74.011	74.004	74.001	0.019
3	73.988	74.024	74.021	74.005	74.002	74.008	0.036
4	74.002	73.996	73.993	74.015	74.009	74.003	0.022
5	73.992	74.007	74.015	73.989	74.014	74.003	0.026
6	74.009	73.994	73.997	73.985	73.993	73.996	0.024
7	73.995	74.006	73.994	74	74.005	74	0.012
8	73.985	74.003	73.993	74.015	73.988	73.997	0.03
9	74.008	73.995	74.009	74.005	74.004	74.004	0.014
10	73.998	74	73.99	74.007	73.995	73.998	0.017
11	73.994	73.998	73.994	73.995	73.99	73.994	0.008
12	74.004	74	74.007	74	73.996	74.001	0.011
13	73.983	74.002	73.998	73.997	74.012	73.998	0.029
14	74.006	73.967	73.994	74	73.984	73.99	0.039

Figure 3.8: Data sheet with process data.

Before computing the desired process capability ratio for a process, the data has to be checked for normality and the process should exhibit control. Once the normality is verified, the $\bar{X}$ and $\bar{R}$ charts are plotted. Plotting of the charts involved computing the upper control limit and the lower control limit for both the charts. The command button ‘X-BAR PLOT’ computes the control limits for the $\bar{X}$ and plots the chart on a separate sheet titled X-Bar chart. The user visually inspects the chart to verify process control. Only after verifying process control can the process capability indices be computed.

The $\bar{X}$ chart is as shown in Figure 3.9 below.

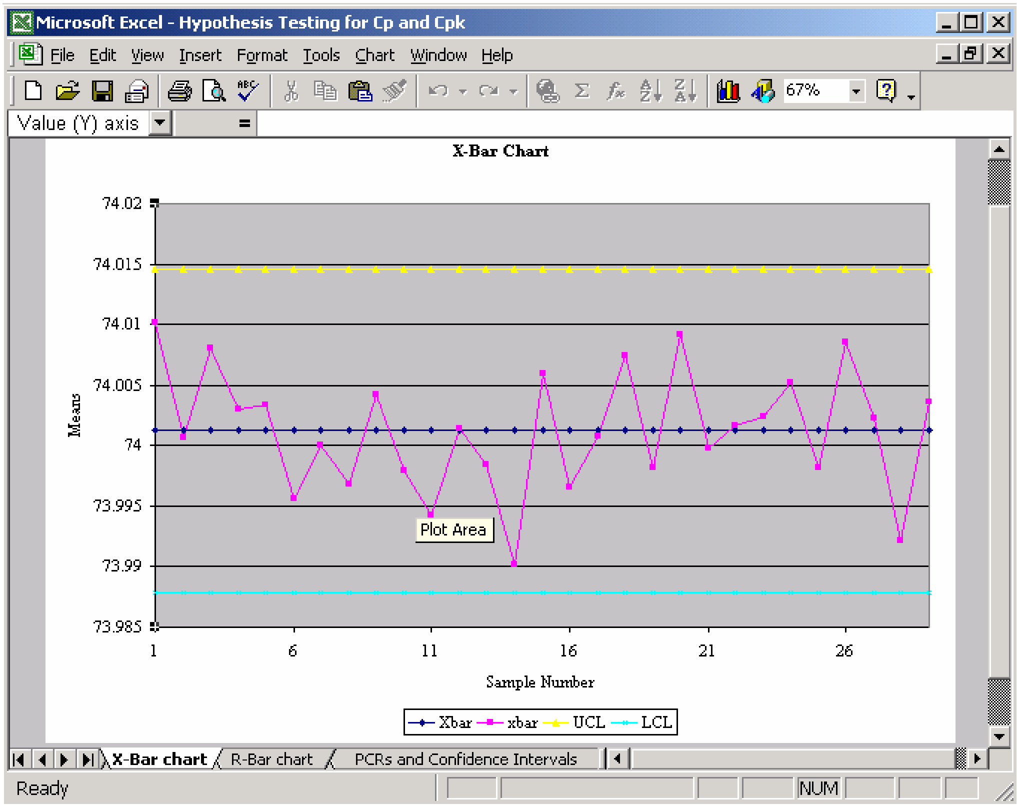


Figure 3.9: $\bar{X}$ chart for process data.

Similarly, the command button ‘R-BAR PLOT’ computes the upper and lower confidence limits for the $\bar{R}$ chart.

The $\bar{R}$ chart is plotted as a separate sheet titled R-Bar chart as shown in Figure 3.10 below.

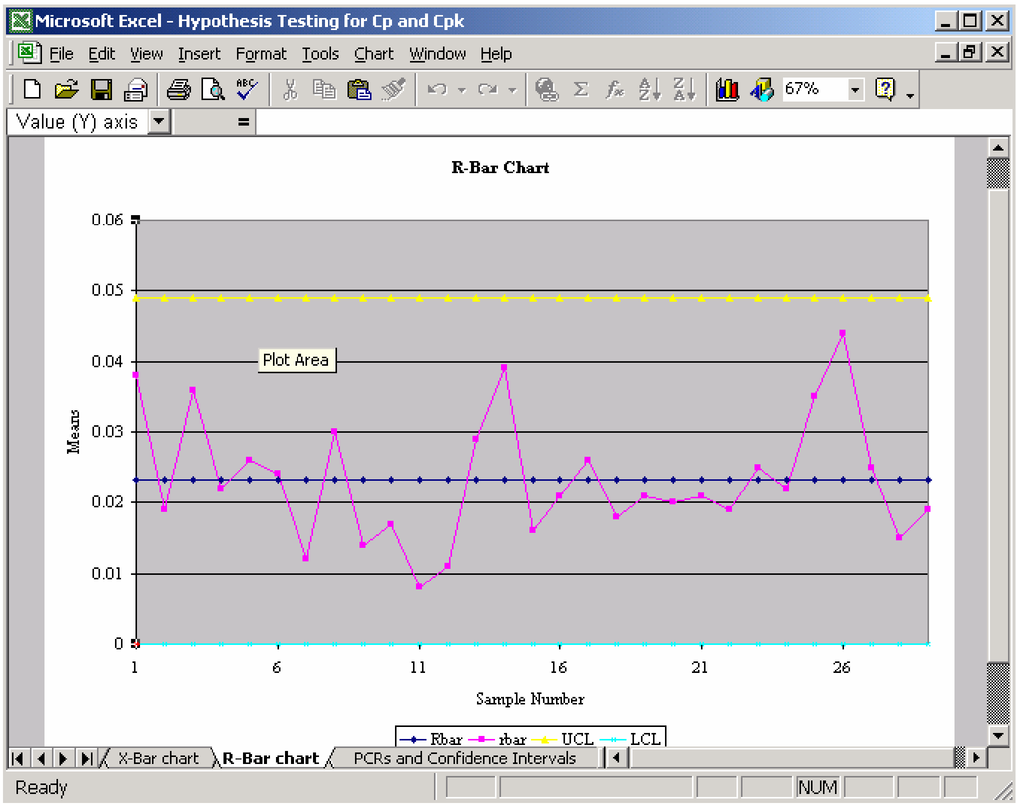
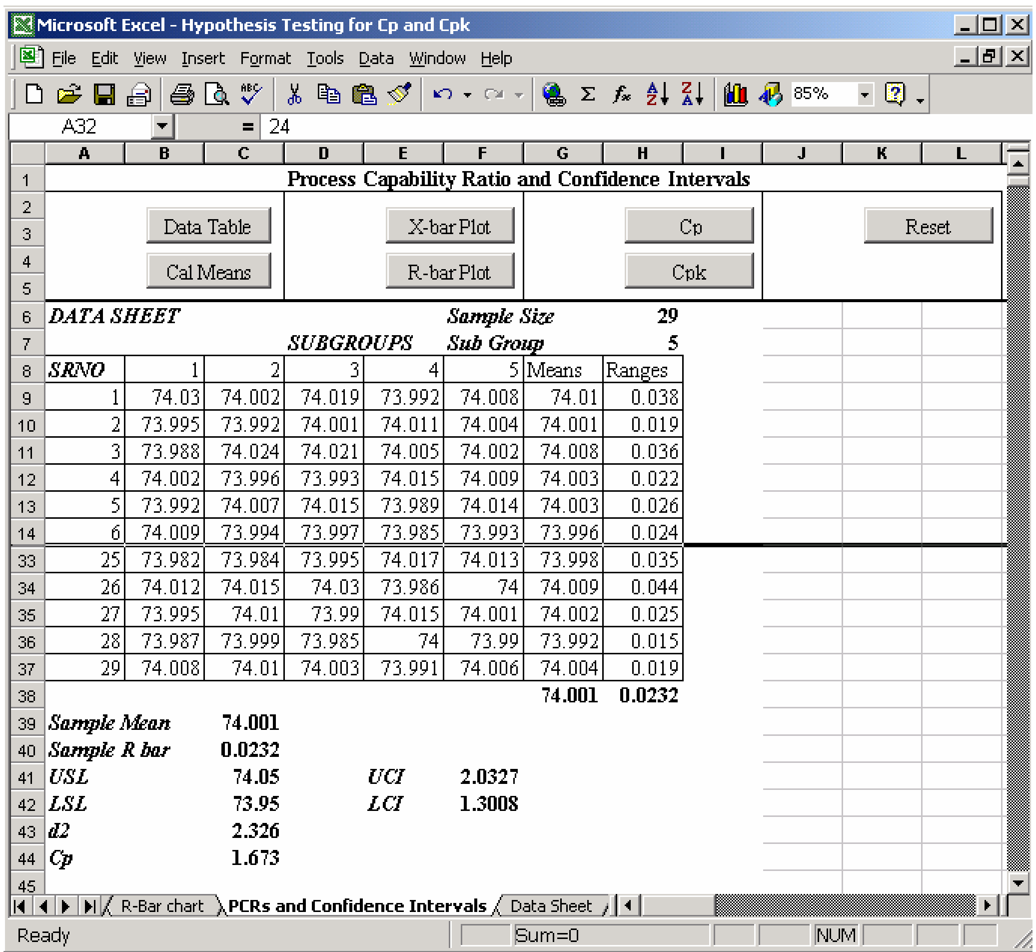


Figure 3.10: $\bar{R}$ chart for process data.

The command buttons 'Cp' and 'Cpk' computes the process capability ratio and calculates the confidence interval. The command buttons 'Cp' and 'Cpk' prompt the user for the upper specification limit (USL) and lower specification limit (LSL) values for the particular process and then compute the process capability index and the upper and lower confidence intervals. The results are displayed on the spreadsheet as shown in Figure 3.11. The command button 'RESET' sets the cells to their initial state and deletes the graphs.



	A	B	C	D	E	F	G	H
6	DATA SHEET					Sample Size		29
7				SUBGROUPS		Sub Group		5
8	SRNO	1	2	3	4	5	Means	Ranges
9	1	74.03	74.002	74.019	73.992	74.008	74.01	0.038
10	2	73.995	73.992	74.001	74.011	74.004	74.001	0.019
11	3	73.988	74.024	74.021	74.005	74.002	74.008	0.036
12	4	74.002	73.996	73.993	74.015	74.009	74.003	0.022
13	5	73.992	74.007	74.015	73.989	74.014	74.003	0.026
14	6	74.009	73.994	73.997	73.985	73.993	73.996	0.024
33	25	73.982	73.984	73.995	74.017	74.013	73.998	0.035
34	26	74.012	74.015	74.03	73.986	74	74.009	0.044
35	27	73.995	74.01	73.99	74.015	74.001	74.002	0.025
36	28	73.987	73.999	73.985	74	73.99	73.992	0.015
37	29	74.008	74.01	74.003	73.991	74.006	74.004	0.019
38							74.001	0.0232
39	Sample Mean		74.001					
40	Sample R bar		0.0232					
41	USL		74.05		UCI	2.0327		
42	LSL		73.95		LCI	1.3008		
43	d2		2.326					
44	Cp		1.673					

Figure 3.11: Process Capability Ratio and Confidence Interval.

Thus, the application allows the user to conduct process capability studies for user specified sampling plans.

Chapter 4: Results and Validation

4.1 Results for the Process Capability Ratio C_p

The program was tested for several combinations to produce sampling plans for both the indices. The results from the program are listed below. Sampling plans for C_p and C_{pk} were developed for similar specifications.

The sample size and cut-off values for the sampling plans for C_p are tabulated in Table 4.1 below.

Table 4.1: Table of Sampling Plans for Process Capability Ratio C_p.

	Sampling Plans					
Sr. No.	α	β	$C_p(low)$	$C_p(high)$	Sample Size, n	Cut-off, c
1	0.05	0.05	1.2	1.6	68	1.41
2	0.02	0.02	1	1.5	54	1.25
3	0.04	0.02	1.1	1.3	261	1.21
4	0.1	0.1	1.2	1.6	42	1.41

The program also plots the operating characteristic curve (OC Curve) for the process capability ratio C_p. The OC curve presents a graphical representation of the sampling plans and is useful for comparing the behavior of different plans. The following Figures (Figure 4.1-Figure 4.4) illustrate the OC curves for the different sampling plans shown in Table 4.1.

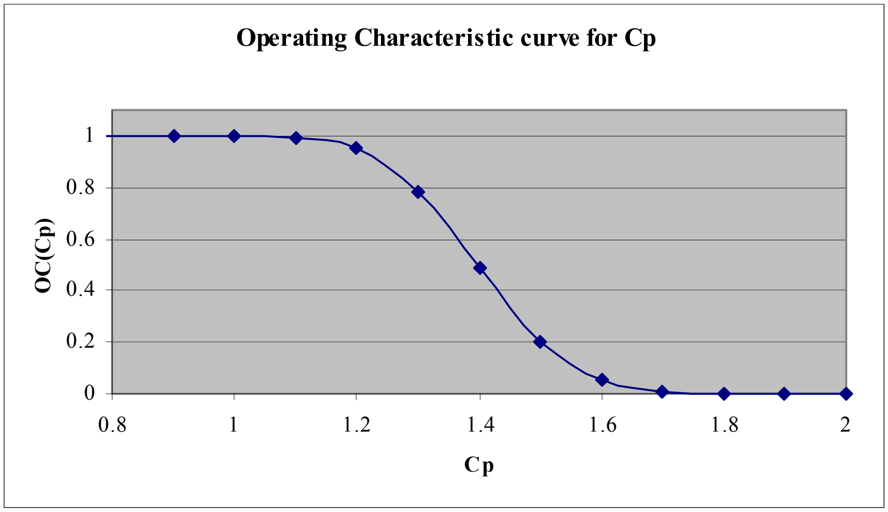


Figure 4.1: OC curve for C_p for sampling plan 1.

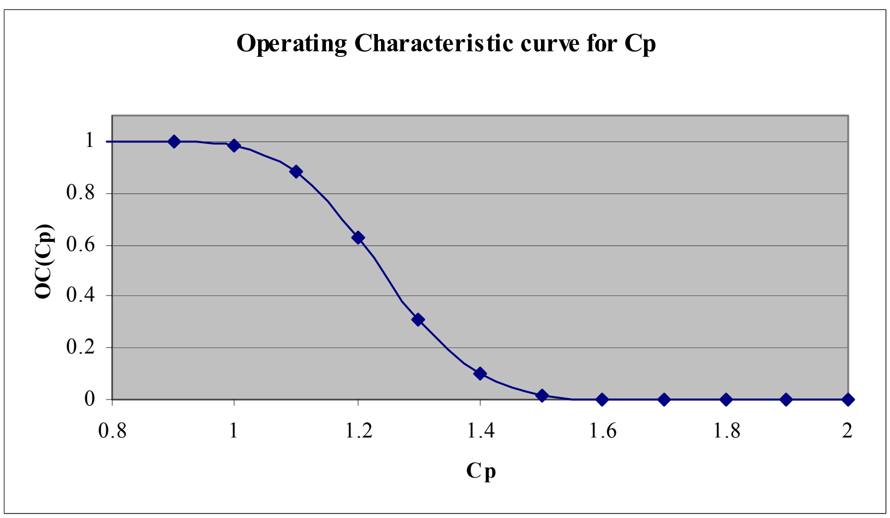


Figure 4.2: OC curve for C_p for sampling plan 2.

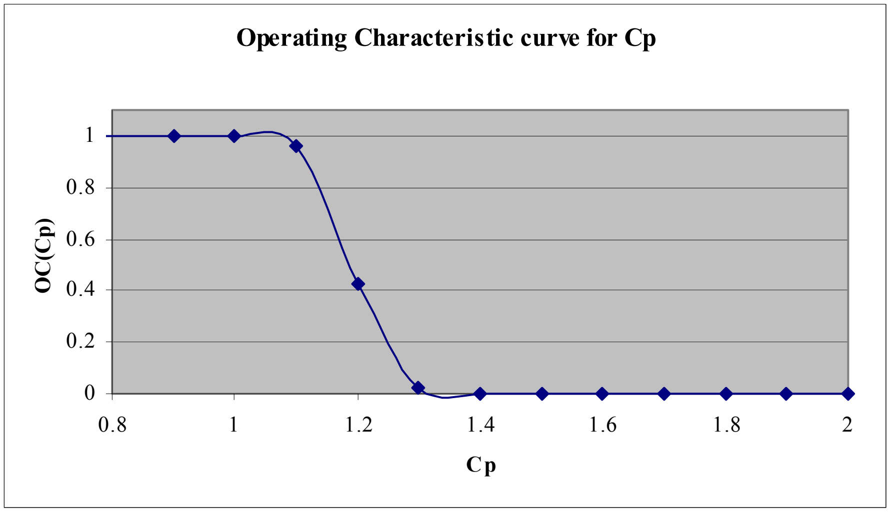


Figure 4.3: OC curve for C_p for sampling plan 3.

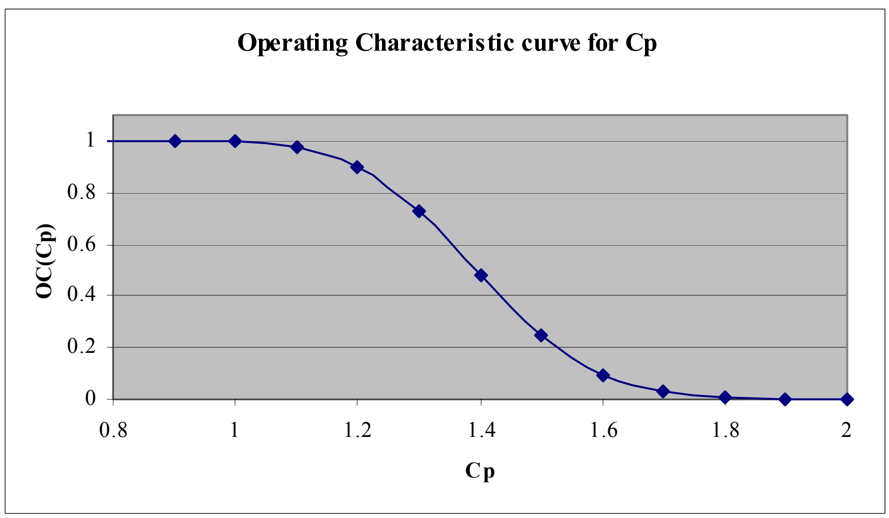


Figure 4.4: OC curve for C_p for sampling plan 4.

Figure 4.5 is a comparison of the OC curves produced by the different sampling plans. The OC curve allows users to evaluate and compare different testing schemes. The OC curve for sampling plan1 shows that at 1.41, OC(C_p) approximately equals 0.40 which implies that there is a 40% chance of judging the process not capable (accepting H_0). The true process capability must be approximately 1.6 before there is only a 5% chance of judging a process not capable using a cut-off value of 1.41.Similarly the OC curves for the other 3 sampling plans can be analyzed to evaluate the testing schemes.

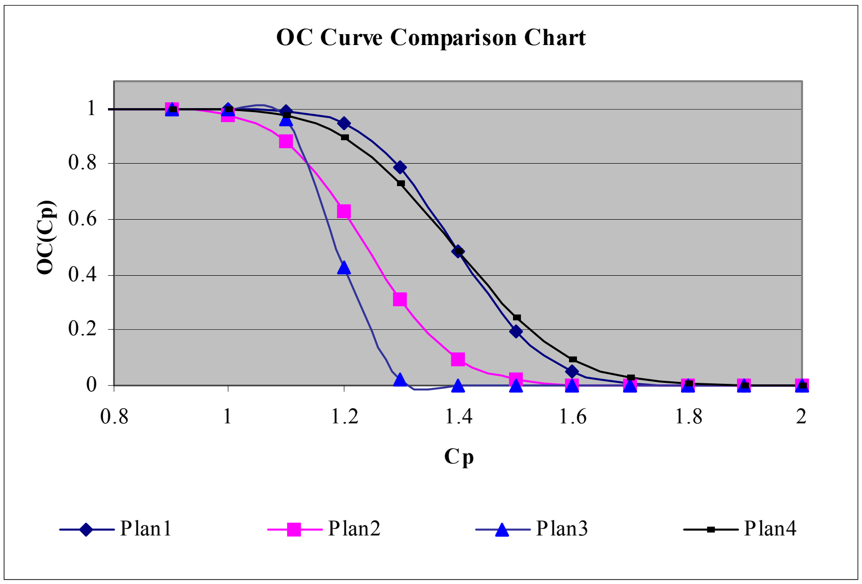


Figure 4.5: OC curve for C_p Comparison Chart.

4.2 Results for the Process Capability Ratio C_{pk}

Sampling plans for the process capability ratio C_{pk} were developed for the same specifications as C_p.

The sample size and cut-off values for the sampling plans for C_{pk} are tabulated in Table 4.2 below.

Table 4.2: Table of Sampling Plans for Process Capability Ratio C_{pk}.

	Sampling Plans					
Sr. No.	α	β	$C_{pk}(low)$	$C_{pk}(high)$	Sample Size, n	Cut-off, c
1	0.05	0.05	1.2	1.6	74	1.37
2	0.02	0.02	1	1.5	61	1.21
3	0.04	0.02	1.1	1.3	302	1.19
4	0.1	0.1	1.2	1.6	45	1.37

4.3 Validation

This section deals with the validation of the program output results. The results for C_p are compared to the results from Kane's paper [3]. A process capability analysis study is conducted for both the indices.

4.3.1. Validation of Results for C_p and C_{pk}

Kane [3] presents an example on how to use his tables to obtain an appropriate sample size and critical value for C_p. In his example:

$\alpha = \beta = 0.05$, $C_p(high) = 1.6$, and $C_p(low) = 1.2$.

Using his table (see Table 3.3) he obtains: sample size = 70, and critical value = 1.37.

Using a critical value of 1.37 and a sample size of 70, there is a 5% risk of judging a process with C_p above 1.6 not capable (i.e., Accepting the null hypothesis

H_0) and a 5% risk of judging a process with C_p below 1.2 as capable (i.e., Rejecting the null hypothesis H_0).

Results from the program: sample size = 68 and critical value = 1.41

The slight difference in the results from the program and Kane's tables is illustrated in the Table 4.3 below.

Table 4.3: Table of Sampling Plan.

For Cp	Sample size	Cutoff
Kane's Tables	70	1.37
Program	68	1.41

The difference may be attributed to the Goalseek function in Excel. To calculate the sample size the following equation is set to zero:

$$\frac{C_p(high)}{C_p(low)} - \sqrt{\frac{\chi^2_{(n-1)}(1-\beta)}{\chi^2_{(n-1)}(\alpha)}} = 0$$

For this particular example Goalseek finds a solution by setting the equation to 0.000l, which is not exactly zero. This produces the slight difference in the sample size. The difference in the cut-off value is significant and this can be attributed to an inaccuracy in Kanes table [3]. The values in the $c / C_p(low)$ column are incorrectly computed. The correct equation for computing $c / C_p(low)$ is given by Chou et al [4]. The error in the values decreases with increase in the sample size.

The table created by Kane (Table 4.4) and the corrected table (Table 4.5) are as shown below.

Table 4.4: Sample size and cut-off value table by Kane [3].

Sample Size	$\alpha = \beta = 0.10$		$\alpha = \beta = 0.05$	
	$\frac{C_p(high)}{C_p(low)}$	$\frac{c}{C_p(low)}$	$\frac{C_p(high)}{C_p(low)}$	$\frac{c}{C_p(low)}$
10	1.88	1.27	2.26	1.37
20	1.53	1.20	1.73	1.26
30	1.41	1.16	1.55	1.20
40	1.34	1.14	1.46	1.18
50	1.30	1.13	1.40	1.16
60	1.27	1.11	1.36	1.15
70	1.25	1.10	1.33	1.14
80	1.23	1.10	1.30	1.13
90	1.21	1.10	1.28	1.12
100	1.20	1.09	1.26	1.11

Table 4.5: Corrected Sample size and cut-off value table.

Sample Size	$\alpha = \beta = 0.10$		$\alpha = \beta = 0.05$	
	$\frac{C_p(high)}{C_p(low)}$	$\frac{c}{C_p(low)}$	$\frac{C_p(high)}{C_p(low)}$	$\frac{c}{C_p(low)}$
10	1.88	1.47	2.26	1.65
20	1.53	1.28	1.73	1.37
30	1.41	1.21	1.55	1.28
40	1.34	1.18	1.46	1.23
50	1.30	1.15	1.40	1.20
60	1.27	1.14	1.36	1.18
70	1.25	1.13	1.33	1.16
80	1.23	1.12	1.30	1.15
90	1.21	1.11	1.28	1.14
100	1.20	1.10	1.26	1.13

4.4 A Capability Analysis Study

A capability analysis study was performed to validate the hypothesis test developed for the process capability ratio C_{pk}. The specifications were similar for the both the indices. This allowed for comparison of the results for C_p and C_{pk}.

The following sampling plan was set to perform a capability analysis study:

For C_p: $\alpha = \beta = 0.1$, $C_p(high) = 1.7$, and $C_p(low) = 1.2$

These specifications produced a sample size of 29 and a cut-off value of 1.48.

For C_{pk}: $\alpha = 0.1$, $\beta = 0.1$ $C_{pk}(high) = 1.7$, and $C_{pk}(low) = 1.2$

These specifications produced a sample size of 31 and a cut-off value of 1.41.

The data for a capability analysis study is included in the appendix. The test data were obtained from Montgomery [2] (pages 187,192). The data represent the inside diameter measurement (mm) on piston rings for an automotive engine produced by a forging process. The specification limits on the piston ring are 74.000 ± 0.05mm.

4.4.1 Validation of data for Capability Analysis Study

Before a capability study is performed the data has to be checked for normality and the process has to exhibit control. The statistical software, Statgraphics was used to plot the frequency histogram

The frequency histogram is often used to examine the data. The histogram displays variation and centering of the data. It also allows for checking normality of the data. The following two figures (Figure 4.6 and Figure 4.7) show the histograms for the data for C_p and C_{pk} analysis.

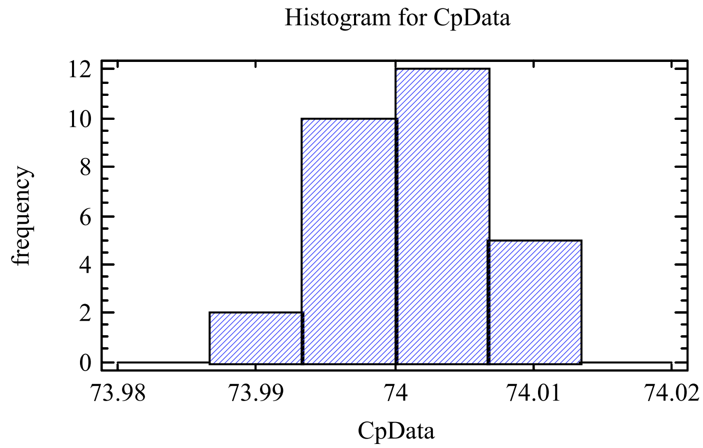


Figure 4.6: Histogram for C_p Data using Statgraphics

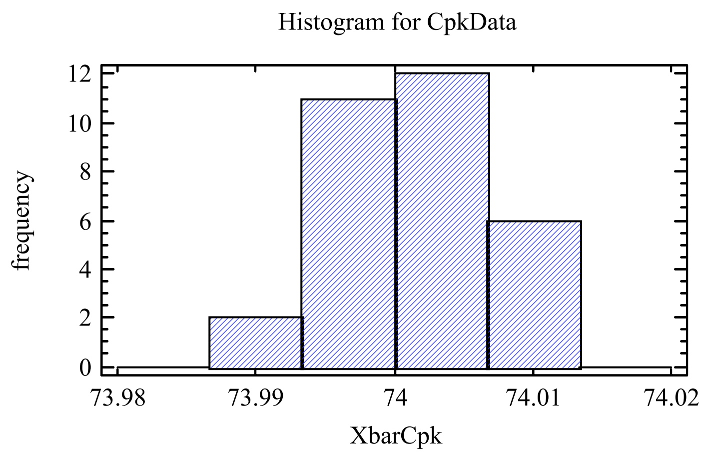


Figure 4.7: Histogram for C_{pk} Data using Statgraphics

The histograms show that the process is approximately normal, centered on nominal and the spread is within the specifications. It can be concluded that the data is normally distributed.

The $\overline{X}$ and R charts are the most commonly recommended control charts for capability purposes [1], because they include both time-to-time variability and random variability. The $\overline{X}$ and R charts and the results are as shown below (Figure 4.8-Figure 4.11 and Table 4.6-Table 4.7). As both the charts exhibit control it can be concluded that the process is in control at the specified levels.

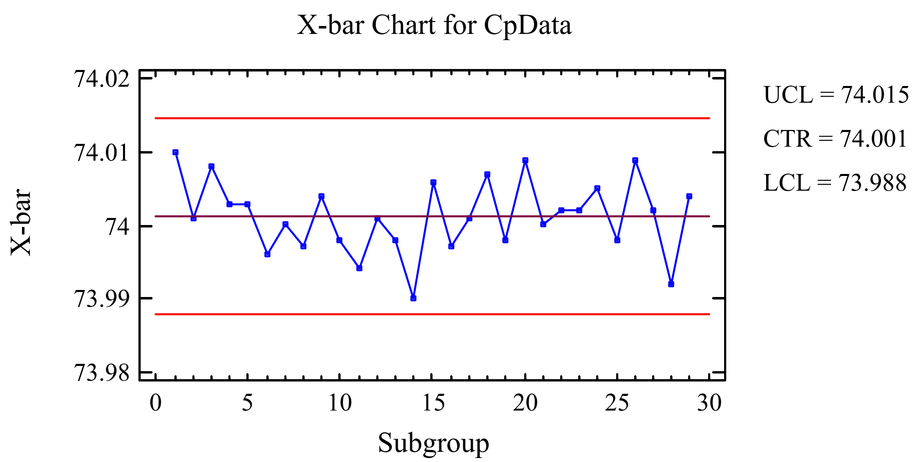


Figure 4.8: X-bar chart for C_p Data using Statgraphics.

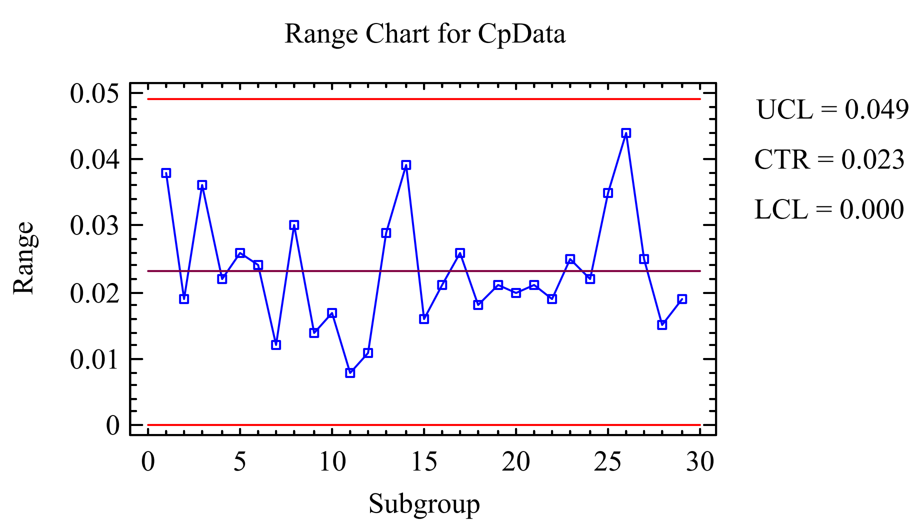


Figure 4.9: Range chart for C_p Data using Statgraphics.

Table 4.6: Summary of results for C_p Data using Statgraphics.

Number of Subgroups: 29			
Subgroup Size: 5			
0 Subgroups excluded			
	UCL: +3.0 sigma	LCL: -3.0 sigma	Centerline
X-bar Chart	74.016000	73.987800	74.001200
R-bar Chart	0.048995	0.000000	0.023172
Estimates			
Process Mean	74.0012		
Process Sigma	0.0099623		
Mean Range	0.0231724		

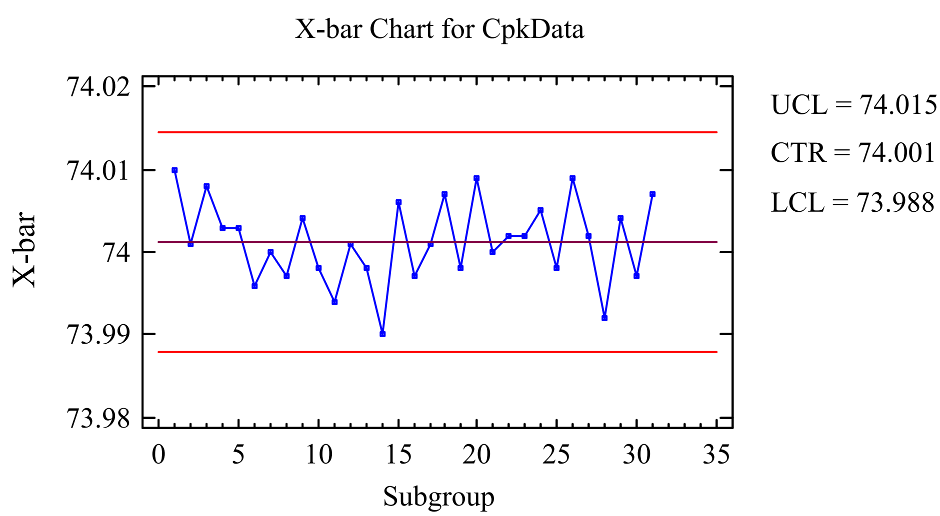


Figure 4.10: X-bar chart for C_{pk} Data using Statgraphics.

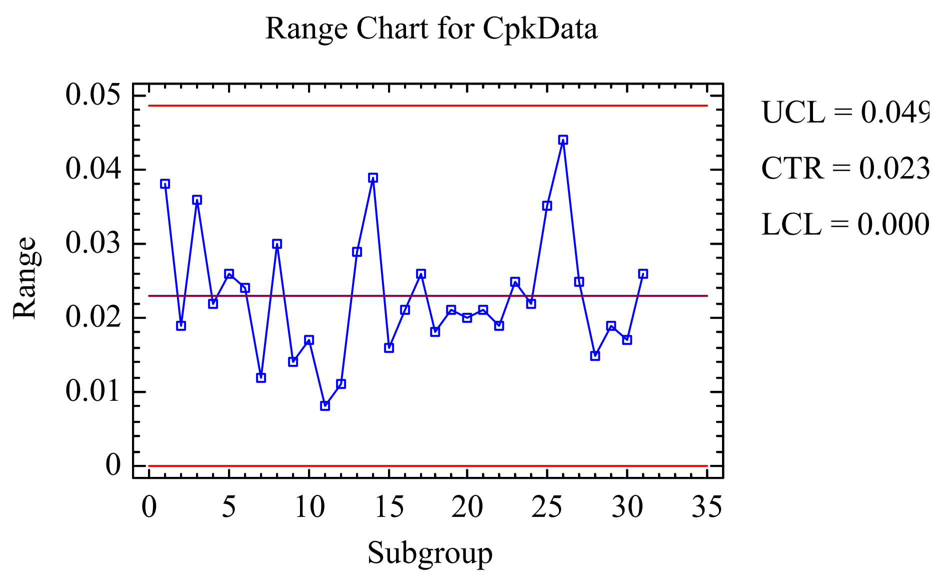


Figure 4.11: Range chart for C_{pk} Data using Statgraphics.

Table 4.7: Summary of results for C_{pk} Data using Statgraphics.

Number of Subgroups: 31			
Subgroup Size: 5			
0 Subgroups excluded			
	UCL: +3.0 sigma	LCL: -3.0 sigma	Centerline
X-bar Chart	74.014600	73.098800	74.001300
R-bar Chart	0.048767	0.000000	0.023065
Estimates			
Process Mean	74.0013		
Process Sigma	0.0099160		
Mean Range	0.0230645		

The following sampling plan was used to perform a capability analysis study:

For C_p : $\alpha = \beta = 0.1$, $C_p(high) = 1.7$, and $C_p(low) = 1.2$

For C_{pk} : $\alpha = 0.1$, $\beta = 0.1$ $C_{pk}(high) = 1.7$, and $C_{pk}(low) = 1.2$

4.4.2 Hypothesis Test and Confidence Interval for C_p

The hypothesis test for C_p is as follows

$$H_0 : \hat{C}_p \leq c$$

$$H_1 : \hat{C}_p > c$$

To demonstrate capability $\hat{C}_p$ must exceed 1.48. This value is computed from the sampling plan for the process capability analysis study for C_p.

$$\hat{C}_p = \frac{USL - LSL}{6\hat{\sigma}}$$

$$\hat{\sigma} = \frac{\overline{R}}{d_2} = 0.02317/2.326 = 0.099$$

$$\hat{C}_p = \frac{74.05 - 73.95}{6(0.01059)} = 1.67$$

Confidence interval for C_p is computed using equation (2.9).

$$1.3 \leq C_p \leq 2.03$$

4.4.3 Hypothesis Test and Confidence Interval for C_{pk}

Hypothesis test for C_{pk}

$$H_0 : \hat{C}_{pk} \leq c_k$$

$$H_1 : \hat{C}_{pk} > c_k$$

To demonstrate capability $\hat{C}_{pk}$ must exceed 1.41. This value is computed from the sampling plan for the process capability analysis study for C_{pk}.

$$\hat{C}_{pk} = \min(\hat{C}_{pl}, \hat{C}_{pu}) = \frac{\min(\hat{\mu} - LSL, USL - \hat{\mu})}{3\hat{\sigma}}$$

$$\hat{\sigma} = \frac{\overline{R}}{d_2} = 0.02306/2.326 = 0.099$$

$$\hat{C}_{pk} = \min(\hat{C}_{pl}, \hat{C}_{pu}) = \frac{\min(\hat{\mu} - LSL, USL - \hat{\mu})}{3\hat{\sigma}}$$

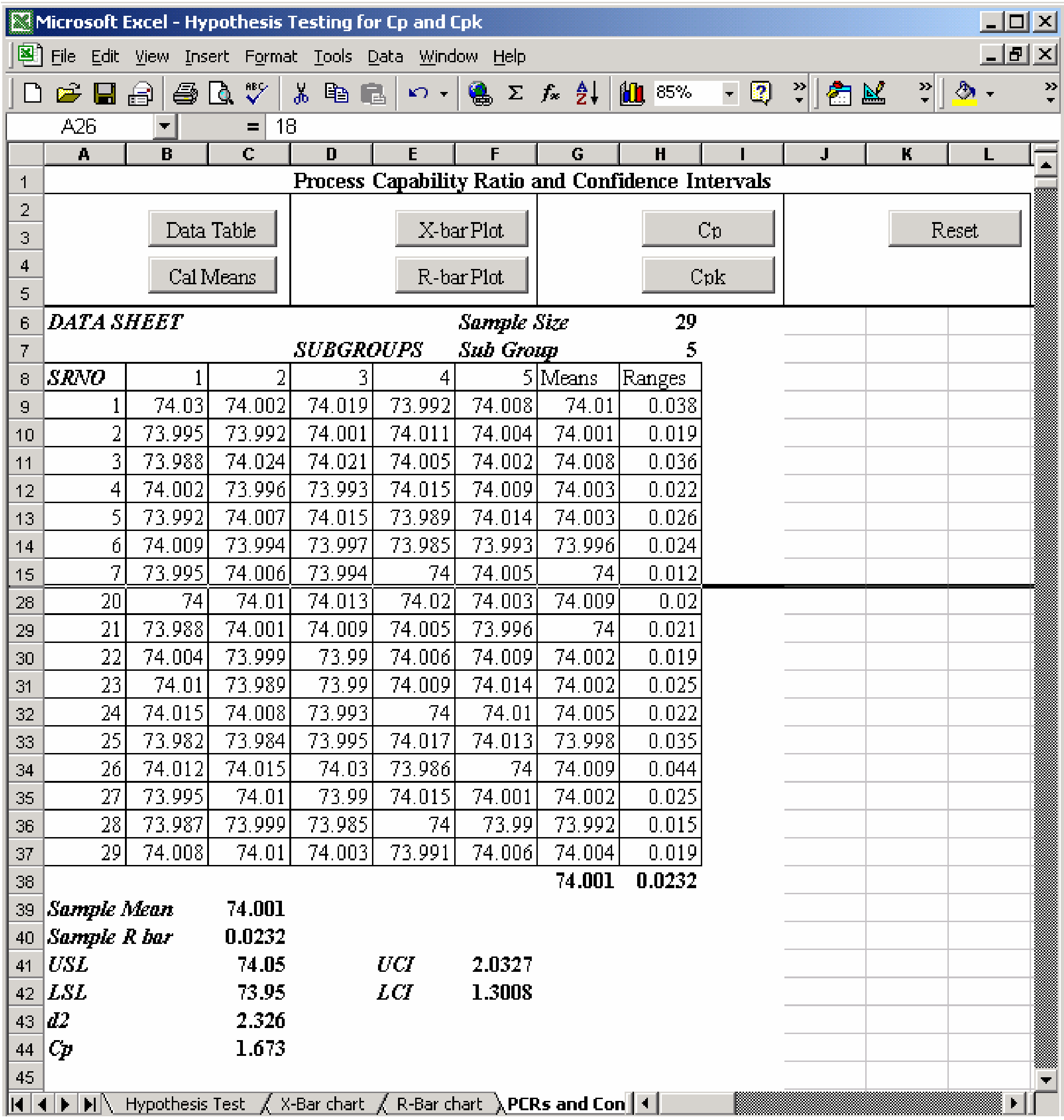
$$\hat{C}_{pk} = 1.63$$

Confidence interval for C_{pk} is computed using equation (2.12).

$$1.23 \leq C_{pk} \leq 2.04$$

4.4.4 Results from the program

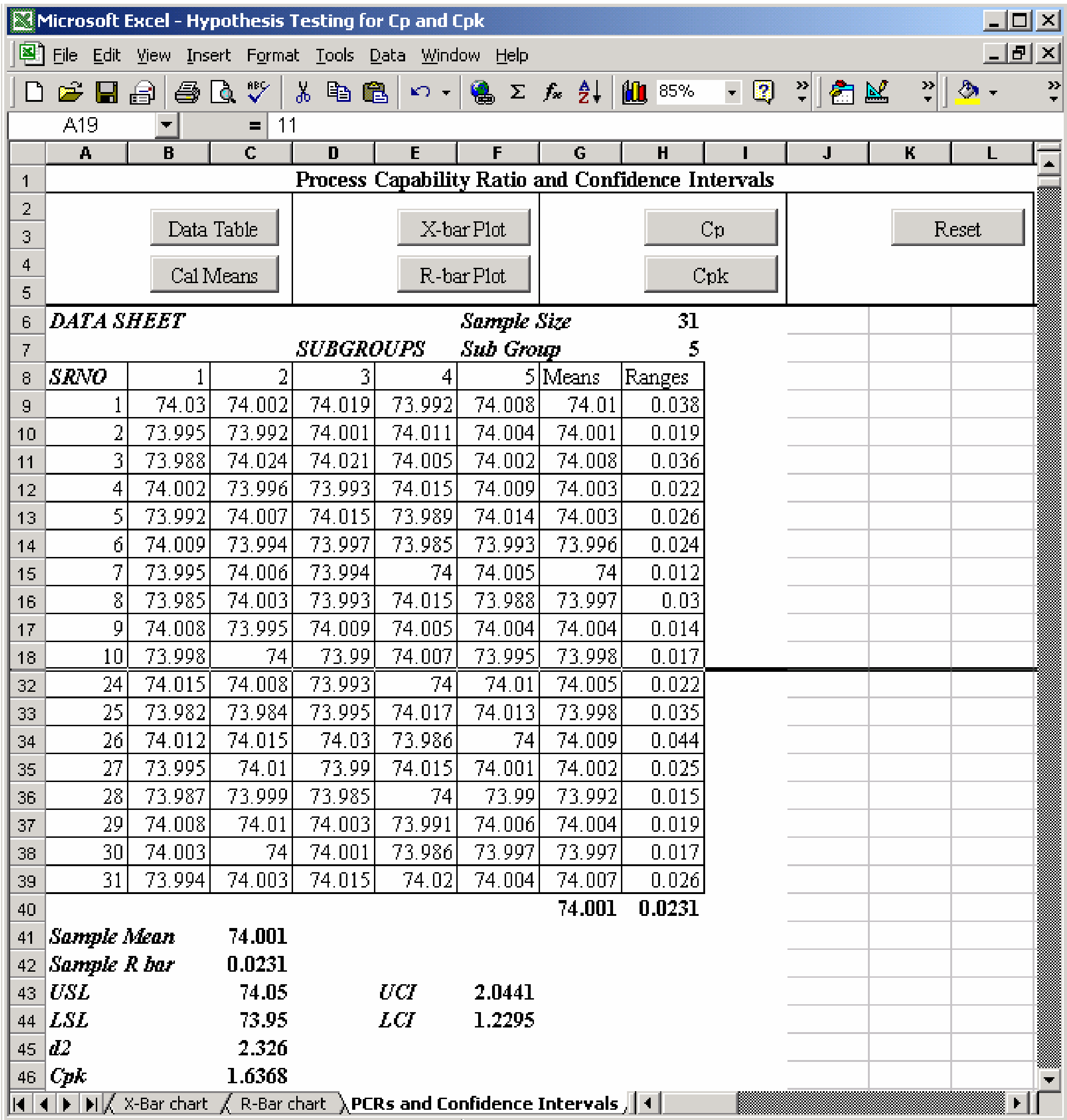
The following figures (Figure 4.12 and Figure 4.13) illustrate the process capability ratio and confidence interval values from the program



Process Capability Ratio and Confidence Intervals

DATA SHEET					*Sample Size*		29
			SUBGROUPS		*Sub Group*		5
SRNO	1	2	3	4	5	Means	Ranges
1	74.03	74.002	74.019	73.992	74.008	74.01	0.038
2	73.995	73.992	74.001	74.011	74.004	74.001	0.019
3	73.988	74.024	74.021	74.005	74.002	74.008	0.036
4	74.002	73.996	73.993	74.015	74.009	74.003	0.022
5	73.992	74.007	74.015	73.989	74.014	74.003	0.026
6	74.009	73.994	73.997	73.985	73.993	73.996	0.024
7	73.995	74.006	73.994	74	74.005	74	0.012
20	74	74.01	74.013	74.02	74.003	74.009	0.02
21	73.988	74.001	74.009	74.005	73.996	74	0.021
22	74.004	73.999	73.99	74.006	74.009	74.002	0.019
23	74.01	73.989	73.99	74.009	74.014	74.002	0.025
24	74.015	74.008	73.993	74	74.01	74.005	0.022
25	73.982	73.984	73.995	74.017	74.013	73.998	0.035
26	74.012	74.015	74.03	73.986	74	74.009	0.044
27	73.995	74.01	73.99	74.015	74.001	74.002	0.025
28	73.987	73.999	73.985	74	73.99	73.992	0.015
29	74.008	74.01	74.003	73.991	74.006	74.004	0.019
						74.001	**0.0232**
Sample Mean		**74.001**					
Sample R bar		**0.0232**					
USL		**74.05**		*UCI*	**2.0327**		
LSL		**73.95**		*LCI*	**1.3008**		
d2		**2.326**					
Cp		**1.673**					

Figure 4.12: Calculation of C_p and confidence interval.



DATA SHEET — *Sample Size* 31

SUBGROUPS — *Sub Group* 5

SRNO	1	2	3	4	5	Means	Ranges
1	74.03	74.002	74.019	73.992	74.008	74.01	0.038
2	73.995	73.992	74.001	74.011	74.004	74.001	0.019
3	73.988	74.024	74.021	74.005	74.002	74.008	0.036
4	74.002	73.996	73.993	74.015	74.009	74.003	0.022
5	73.992	74.007	74.015	73.989	74.014	74.003	0.026
6	74.009	73.994	73.997	73.985	73.993	73.996	0.024
7	73.995	74.006	73.994	74	74.005	74	0.012
8	73.985	74.003	73.993	74.015	73.988	73.997	0.03
9	74.008	73.995	74.009	74.005	74.004	74.004	0.014
10	73.998	74	73.99	74.007	73.995	73.998	0.017
24	74.015	74.008	73.993	74	74.01	74.005	0.022
25	73.982	73.984	73.995	74.017	74.013	73.998	0.035
26	74.012	74.015	74.03	73.986	74	74.009	0.044
27	73.995	74.01	73.99	74.015	74.001	74.002	0.025
28	73.987	73.999	73.985	74	73.99	73.992	0.015
29	74.008	74.01	74.003	73.991	74.006	74.004	0.019
30	74.003	74	74.001	73.986	73.997	73.997	0.017
31	73.994	74.003	74.015	74.02	74.004	74.007	0.026
						74.001	**0.0231**

Sample Mean	**74.001**		
Sample R bar	**0.0231**		
USL	**74.05**	*UCI*	**2.0441**
LSL	**73.95**	*LCI*	**1.2295**
d2	**2.326**		
Cpk	**1.6368**		

Figure 4.13: Calculation of C_{pk} and confidence interval.

Both $\hat{C}_p$ and $\hat{C}_{pk}$ have values greater than the cut-off values for their respective sampling plans. Thus, the process exhibits control at the required specifications.

The confidence intervals on both C_p and C_{pk} are extremely wide. This is because of the moderately small sample size involved. In this case the confidence interval reveals little about the capability of the process. A hypothesis test becomes a useful tool to qualify the results.

Chapter 5: Conclusions

5.1 Conclusions

This thesis has developed a hypothesis test for the process capability ratio C_{pk}. The proper utilization of estimation qualification is important in the case of C_{pk} as it is widely used in managerial decisions. Qualification of an estimate is provided in the form of confidence intervals or hypothesis testing. The hypothesis test developed in this thesis will provide practitioners with an additional method for estimate qualification.

The program provides an interface for users to develop sampling plans for C_{pk} as well as C_p. This eliminates the need for tables as developed by Kane. The program can calculate sample sizes and cut-off values for user-defined data. The program also allows the user to perform process capability analysis studies. It allows the user to check for process control using the $\bar{X}$ and $\bar{R}$ charts and then computes the required index and confidence interval. The sampling plans developed for both the indices can be utilized for capability analysis studies as shown in the validation example.

Due to the nature of development of capability indices several precautions need to be taken when using the indices. The data from the process should be normal and the process should be in control. C_p and C_{pk} are not designed to handle non-normal data. There are specific indices that address non-normality in the data. The hypothesis test for C_{pk} has been developed from an approximate confidence interval. This fact should be taken into account when interpreting the result from such a test. However, if the above mentioned issues are kept in mind the hypothesis test will provide practitioners with an additional estimate qualification tool.

5.2 Future Scope

Considerable enhancements can be made to the user interface developed in this thesis. Added functionality can allow the user to check for normality of process data. At present the program only allows the user to plot the $\bar{X}$ and $\bar{R}$ charts to verify process control. This thesis deals with the OC curve for C_p. Further research could lead to the development of an OC curve for C_{pk} and incorporating it in the program. This research concentrated on the most commonly used indices C_p and C_{pk}. There are a number of process capability indices in use today. Some of them deal with special cases of data and are highly specialized and other deal with limitations of C_p and C_{pk}. Developing a hypothesis test for some of the new indices will be a natural extension to this thesis.

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Appendix A: Data for Process Capability Study.

Test data for process capability analysis ratio C_p

The data represents inside diameter measurements (mm) on piston rings for an automotive engine produced by a forging process. The specification limits on the piston ring are 74.000 ± 0.05mm. The data has 29 samples, each of size 5.

Sr No	Observations				
1	74.030	74.002	74.019	73.992	74.008
2	73.995	73.992	74.001	74.011	74.004
3	73.988	74.024	74.021	74.005	74.002
4	74.002	73.996	73.993	74.015	74.009
5	73.992	74.007	74.015	73.989	74.014
6	74.009	73.994	73.997	73.985	73.993
7	73.995	74.006	73.994	74.000	74.005
8	73.985	74.003	73.993	74.015	73.988
9	74.008	73.995	74.009	74.005	74.004
10	73.998	74.000	73.990	74.007	73.995
11	73.994	73.998	73.994	73.995	73.990
12	74.004	74.000	74.007	74.000	73.996
13	73.983	74.002	73.998	73.997	74.012
14	74.006	73.967	73.994	74.000	73.984
15	74.012	74.014	73.998	73.999	74.007
16	74.000	73.984	74.005	73.998	73.996
17	73.994	74.012	73.986	74.005	74.007
18	74.006	74.010	74.018	74.003	74.000
19	73.984	74.002	74.003	74.005	73.997
20	74.000	74.010	74.013	74.020	74.003
21	73.988	74.001	74.009	74.005	73.996
22	74.004	73.999	73.990	74.006	74.009
23	74.010	73.989	73.990	74.009	74.014
24	74.015	74.008	73.993	74.000	74.010
25	73.982	73.984	73.995	74.017	74.013
26	74.012	74.015	74.030	73.986	74.000
27	73.995	74.010	73.990	74.015	74.001
28	73.987	73.999	73.985	74.000	73.990
29	74.008	74.010	74.003	73.991	74.006

Test data for process capability analysis ratio C_{pk}

The data has 31 samples, each of size 5.

Sr No	Observations				
1	74.030	74.002	74.019	73.992	74.008
2	73.995	73.992	74.001	74.011	74.004
3	73.988	74.024	74.021	74.005	74.002
4	74.002	73.996	73.993	74.015	74.009
5	73.992	74.007	74.015	73.989	74.014
6	74.009	73.994	73.997	73.985	73.993
7	73.995	74.006	73.994	74.000	74.005
8	73.985	74.003	73.993	74.015	73.988
9	74.008	73.995	74.009	74.005	74.004
10	73.998	74.000	73.990	74.007	73.995
11	73.994	73.998	73.994	73.995	73.990
12	74.004	74.000	74.007	74.000	73.996
13	73.983	74.002	73.998	73.997	74.012
14	74.006	73.967	73.994	74.000	73.984
15	74.012	74.014	73.998	73.999	74.007
16	74.000	73.984	74.005	73.998	73.996
17	73.994	74.012	73.986	74.005	74.007
18	74.006	74.010	74.018	74.003	74.000
19	73.984	74.002	74.003	74.005	73.997
20	74.000	74.010	74.013	74.020	74.003
21	73.988	74.001	74.009	74.005	73.996
22	74.004	73.999	73.990	74.006	74.009
23	74.010	73.989	73.990	74.009	74.014
24	74.015	74.008	73.993	74.000	74.010
25	73.982	73.984	73.995	74.017	74.013
26	74.012	74.015	74.030	73.986	74.000
27	73.995	74.010	73.990	74.015	74.001
28	73.987	73.999	73.985	74.000	73.990
29	74.008	74.010	74.003	73.991	74.006
30	74.003	74.000	74.001	73.986	73.997
31	73.994	74.003	74.015	74.020	74.004